Information Aggregation in Competitive Markets*

Lucas Siga[†] Maximilian Mihm[‡]

June 27, 2020

Abstract

We study when equilibrium prices can aggregate information in an auction market with a large population of traders. Our main result identifies a property of information—the *betweenness property*—that is both necessary and sufficient for information aggregation. The characterization provides novel predictions about equilibrium prices in complex, multidimensional environments.

1 Introduction

When do market prices aggregate information? This question is central to understanding a market economy where information about unknown fundamentals is dispersed over many market participants, and prices are often the primary channel whereby information is aggregated and transmitted in the economy.

In this paper, we study information aggregation in a competitive market with an infinite population of privately informed traders. Trade occurs through an auction mechanism that closely resembles the call market used, for instance, to set daily opening prices on the New York Stock Exchange. There is a fixed supply of assets

*We thank Nageeb Ali, Vladimir Asriyan, Paulo Barelli, Pablo Beker, Larry Blume, Olivier Bochet, Aaron Bodoh-Creed, Vince Crawford, Alessandro Citanna, Mehmet Ekmekci, Vijay Krishna, Mark Machina, Cesar Martinelli, Samreen Malik, Larry Samuelson, Tom Sargent, Ricardo Serrano-Padial, Paolo Siconolfi, Ran Spiegler, Jeroen Swinkels, Juuso Välimäki, and Joel Watson for helpful suggestions. We also thank the coeditor, Simon Board, and six anonymous referees for their comments, which significantly improved the paper. Special thanks to Joel Sobel for his invaluable guidance at various stages of this project and to Ayelen Banegas for our many insightful discussions.

[†]Division of Social Science, New York University Abu Dhabi. Email: lucas.siga@nyu.edu.

[‡]Division of Social Science, New York University Abu Dhabi. Email: max.mihm@nyu.edu.

and the value of a unit of asset is common, but unknown. After observing private signals, traders submit sealed bids and an auctioneer determines the market-clearing price. With their bids, traders affect their allocation, but the large population implies that individual traders have negligible impact on prices. Accordingly, our model encompasses the key price-taking assumption of competitive equilibrium models, but with an explicit price formation process based on strategic auction models.[1]

Our main result characterizes the information environments where equilibrium prices can aggregate information. On one hand, we show that prices *can* aggregate information even in complex environments where the prior auction literature makes no predictions about the information efficiency of prices. On the other hand, we establish limitations of the auction-mechanism by identifying environments where auction prices *cannot* implement a fully-revealing rational expectations equilibrium.

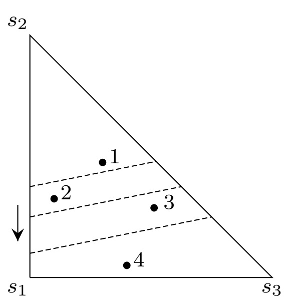
The environments we consider have a finite set of states Ω and signals S, with a joint-probability distribution P on $\Omega \times S$. A unit of asset has common-value $v(\omega) \geq 0$ in state ω. The state is unknown, but traders independently draw a signal from the conditional distribution over signals $P_\omega \in \Delta$. Traders then submit bids from a compact interval B, and the distribution of bids determines a market-clearing price. The key primitives of the environment are the value function $v : \Omega \to B$ and the information structure $\{P_\omega : \omega \in \Omega\}$.

For such environments, we show that equilibrium prices can aggregate information if and only if there is a *quasi-linear* function $V : \Delta \to \mathbb{R}$ that is *monotone in values*: $v(\omega) > v(\omega')$ implies $V(P_\omega) > V(P_{\omega'})$. Quasi-linear functions are a generalization of linear functions: while linear functions are concave and convex, quasi-linear functions are quasi-concave and quasi-convex. In particular, they are characterized by a betweenness condition, $V(x) \geq V(y)$ implies $V(x) \geq V(\theta x + (1-\theta)y) \geq V(y)$, which means that level sets are linear (upper and lower contour sets are convex). When there is a quasi-linear function that is monotone in values, we therefore say that the environment satisfies the *betweenness property.*

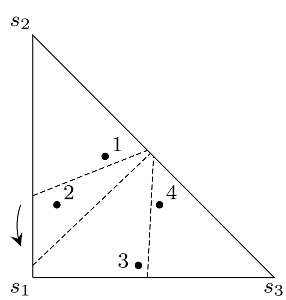
The betweenness property has a simple geometric interpretation. Figure 1 provides an illustration with four states and three signals. A state is represented by a point in the simplex, which indicates the conditional distribution over signal and the value.

[1] We therefore combine insights from Aumann (1964, p.39), who argues that "*a mathematical model appropriate to the intuitive notion of perfect competition must contain infinitely many participants*," and Milgrom (1981, p.923), who argues that "*to address seriously such questions as how do prices come to reflect information...one needs a theory of how prices are formed.*"

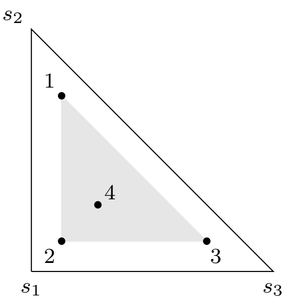
The level sets of a linear function are straight and parallel; for a quasi-linear function, level sets are straight but not necessarily parallel. In panel (a), there is a linear function that is monotone in values, and so the betweenness property is satisfied. In panel (b), a linear function cannot be monotone in values, but there is a quasi-linear function that is monotone values, and so the betweenness property is satisfied. In panel (c), a quasi-linear function cannot be monotone in values because the high value state is in the convex hull of the low value states.



(a) Linearity



(b) Betweenness



(c) Failure of betweenness

Figure 1: Betweenness property with four states and three signals.

We identify three features of the market, which establish that the betweenness property is necessary and sufficient for information aggregation. The first feature reflects a general limitation of the auction-mechanism: *any* strategy-profile σ induces a price function $\tilde{p}_\sigma : \Delta \to B$ that is quasi-linear. In particular, since auction-prices depend on a single quantile of the bid distribution, average conditional distributions always induce average prices (i.e., prices satisfy betweenness). The second feature reflects the flexibility of the auction-mechanism: for *any* quasi-linear function $V : \Delta \to B$, there is a strategy-profile σ so that $\tilde{p}_\sigma(x) = V(x)$ for all $x \in \Delta$. In particular, beyond quasi-linearity, there are no other general constraints on prices from the auction-mechanism. The last feature reflects how competition disciplines prices: an *equilibrium* aggregates information if and only if prices equal values, otherwise traders have arbitrage opportunities.

It is then straightforward to see why the betweenness property is necessary for information aggregation. The price function $\tilde{p}_\sigma : \Delta \to B$ for any strategy-profile σ is quasi-linear. Moreover, if σ is an equilibrium, no-arbitrage implies that $\tilde{p}_\sigma(P_\omega) = v(\omega)$ for every state ω. The price function is therefore also monotone in values, which is exactly what the betweenness property posits. On the other hand, when the betweenness property is satisfied, there is a quasi-linear function $V : \Delta \to \mathbb{R}$ that is monotone in values, and can be normalized to have range B. As a result, there is a strategy-profile σ for which the price-function $\tilde{p}_\sigma$ is monotone in values. It is then always possible to re-scale bids to ensure that the price equals the value in every state, and traders have no further arbitrage opportunities. The betweenness property is therefore also sufficient for information aggregation.

Overall, our analysis highlights both the power and limitations of a competitive auction market. On one hand, we show that the betweenness property is generic as long as there are at least as many signals as states. This illustrates the power of market prices in rich (but potentially very noisy) information environments. On the other hand, when there are more states than signals, we show that the betweenness property is restrictive. While a fully-revealing REE always exists, it generally cannot be implemented with equilibrium auction prices. This highlights limitations of the market when prices must distinguish between many values with limited signals, and trading strategies must be measurable with respect to private information.

We view these results as especially relevant in multidimensional environments. The prior literature on information aggregation in auctions makes extensive use of the monotone likelihood ratio property (MLRP), which imposes a linear order on signals that reduces the dimensionality of the information environment. By focussing on ordinal properties of the *distribution* over signals, rather than the signals themselves, we do not restrict the dimensionality of the states or signals. To illustrate, we consider a class of environments where states have multiple inputs and signals are specific to inputs. A signal then conveys information for only one dimension of the asset's value, and traders must rely on prices to aggregate the fragmented information diffused in the marketplace. We show that the MLRP is never satisfied in such environments, yet equilibrium prices in a competitive auction can aggregate information generically whenever there are as many signals as states for each input.

The paper is organized as follows. In Section 2, we discuss an example to highlight some key insights of our analysis. Section 3 discusses related literature. Section 4 defines the betweenness property and describes the market. Section 5 presents our main result, provides a proof sketch, and discusses various extensions. Section 6 presents our genericity results, and Section 7 provides the application to multi-input environments. Section 8 concludes. Formal proofs are given in the Appendix.

2 Example

To fix idea, consider a market with a mass one of traders and a mass one-half of assets. The common-value of a unit of asset depends on two independent inputs, A and B. For instance, the value of the asset could reflect the real returns from an investment in two different sectors, or the yields of a commodity in two different locations.

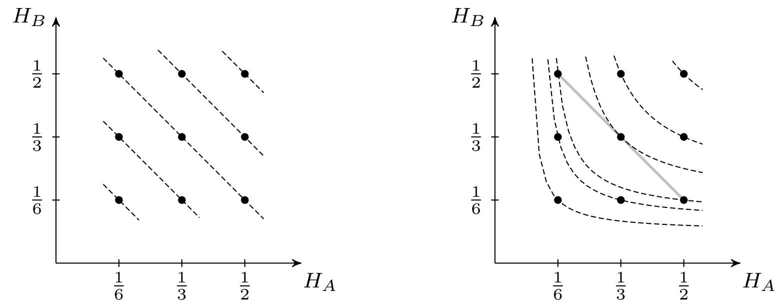
Let ω_c be the value of input c and suppose that each input can take three possible values: $\omega_c \in \{1, 2, 3\}$. The value of the asset is a *constant elasticity of substitution* (CES) aggregate of the two inputs: for some $\alpha \in (0, 1)$ and $\rho \leq 1$,

$$v(\omega_A, \omega_B) = \begin{cases} \left(\alpha\omega_A^{\rho} + (1-\alpha)\omega_B^{\rho}\right)^{\frac{1}{\rho}} & \text{if } \rho \neq 0 \\ \omega_A^{\alpha}\,\omega_B^{1-\alpha} & \text{if } \rho = 0 \end{cases}.$$

The parameter α measures the relative impact of the two inputs on the value: when $\alpha = \frac{1}{2}$, the value is symmetric, and input A has greater impact as α increases. The parameter ρ measures the complementarity between inputs: when $\rho = 1$, inputs are perfect substitutes, and a smaller ρ reflects a higher degree of complementarity.

Traders are ex-ante identical, but receive specialized information (e.g, by industry or region): their signals are informative about one input but convey no information about the other. In particular, suppose there are two signals for each input: a low signal L_c and a high signal H_c. A trader is equally likely to receive a signal for each input. In state (ω_A, ω_B), conditional on a signal for input c, the probability of H_c is $\frac{\omega_c}{3}$. As such, a high signal is more likely when input c has a higher value (see Figure 2). While the MLRP is satisfied for each input, information is fragmented and the overall information structure does not satisfy the MLRP (see Proposition 4 in Section 7).

In a REE, trader's augment their private signals with information conveyed by equilibrium prices. This market always has a fully-revealing REE because, if the equilibrium price equals the value, traders can ignore their own signals. However, it is therefore also unclear where prices originate, or how they incorporate information (Hellwig, 1980; Milgrom, 1981). The auction literature addresses this problem by providing a complete description of the trading mechanism. However, in order to establish an equilibrium in monotone bidding strategies, this literature imposes that signals satisfy the MLRP. Nothing is known about the information conveyed by auction prices when the MLRP is not satisfied. Our model does not impose the MLRP, and



(a) $\rho = 1$ and $\alpha = 0.5$ (b) $\rho = -1$ and $\alpha = 0.5$

Figure 2: CES value function with symmetric inputs.

We can depict the environment by the probability that a trader receives one of the high signals. Dashed lines represent iso-value lines and points represent states; ρ determines the curvature of the iso-value lines, and α determines the rotation.

provides a way to characterize when information can be aggregated in an equilibrium where traders condition bids only on their private signals. We highlight some features of the model by addressing three questions.

Can prices aggregate information? First, suppose inputs are symmetric ($\alpha = \frac{1}{2}$) and perfect substitutes ($\rho = 1$). Then we argue that equilibrium prices *can* aggregate information even though signals do not satisfy the MLRP.

To illustrate, consider the following strategy: when trader i receives a high signal, she randomizes over bids with the cumulative distribution function $F_H(b) = \min\{\frac{1}{6}b, 1\}$; when she receives a low signal, she randomizes with the cumulative distribution function $F_L(b) = \min\{\frac{1}{2} + \frac{1}{6}b, 1\}$. When all traders follow this strategy, we can appeal to the LLN to describe aggregate demand.

In state $(1,1)$, the probability of a high signal is $\frac{1}{3}$ for both inputs, and the probability of a low signal is $\frac{2}{3}$. Therefore, one-third of the traders receive a high signal and two-thirds receive a low signal. Moreover, since traders randomize independently, a proportion $1-\frac{1}{6}b$ of the traders with high signals, and a proportion $(\frac{1}{2}-\frac{1}{6}b)$ with low signals, submit a bid greater than b. The total mass of traders submitting a bid greater than b is therefore $\frac{1}{3}(1-\frac{1}{6}b)+\frac{2}{3}(\frac{1}{2}-\frac{1}{6}b)=\frac{4}{6}-\frac{1}{6}b$. We can therefore interpret $\frac{4}{6}-\frac{1}{6}b$ as the aggregate demand in state $(1,1)$. The market-clearing price p^* must equate aggregate demand with supply (i.e., solve $\frac{4}{6}-\frac{1}{6}p^*=\frac{1}{2}$), which implies $p^*_{(1,1)}=1$.

Similarly, in states $(2,1)$ and $(1,2)$, half of the traders receive high signals and half receive low signals, aggregate demand is therefore $\frac{1}{2}(1-\frac{1}{6}b)+\frac{1}{2}(\frac{1}{2}-\frac{1}{6}b)=\frac{3}{4}-\frac{1}{6}b$, and the market-clearing price is $p^*_{(2,1)}=p^*_{(1,2)}=\frac{3}{2}$. In states $(3,1)$, $(2,2)$ and $(1,3)$, two-thirds of the traders receive high signals and one-third receive low signals, aggregate demand is therefore $\frac{2}{3}(1-\frac{1}{6}b)+\frac{1}{3}(\frac{1}{2}-\frac{1}{6}b)=\frac{5}{6}-\frac{1}{6}b$, and $p^*_{(3,1)}=p^*_{(2,2)}=p^*_{(1,3)}=2$. In states $(3,2)$ and $(2,3)$, five-sixth of the traders receive high signals and one-sixth receive low signals, aggregate demand is therefore $\frac{5}{6}(1-\frac{1}{6}b)+\frac{1}{6}(\frac{1}{2}-\frac{1}{6}b) =\frac{11}{12}-\frac{1}{6}b$, and $p^*_{(3,2)}=p^*_{(2,3)}=\frac{5}{2}$. In state $(3,3)$, all traders receive high signals, aggregate demand is therefore $1-\frac{1}{6}b$, and $p^*_{(3,3)}=3$ (see Figure 3).

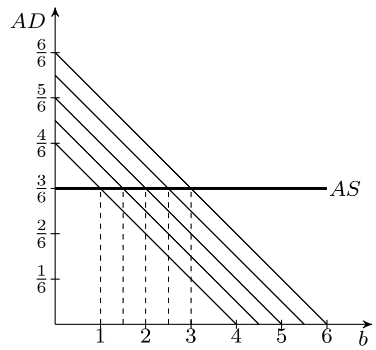


Figure 3: Aggregate demand and supply.

As a result, the market-clearing price is equal to the value in every state. While individual traders have noisy signals, market prices aggregate information. Moreover, the strategy is an equilibrium because individual traders cannot impact prices and therefore make zero profits from any deviation. The equilibrium therefore has all the properties of a fully-revealing REE, except that individual strategies are measurable with respect to individual signals.

Can prices always aggregate information? Next, suppose signals are symmetric ($\alpha = \frac{1}{2}$) but are not perfect substitutes ($\rho < 1$). In that case, we argue that equilibrium prices *cannot* aggregate information even when there is a fully-revealing REE.

To illustrate, consider any strategy-profile, and let $1 - F_s(b)$ be the proportion of traders who submit a bid greater than b when they receive signal s.[2] Aggregate demand in state $(3,1)$ is therefore $1-\left(\frac{1}{2}F_{H_A}(b)+\frac{1}{6}F_{H_B}(b)+\frac{1}{3}F_{L_B}(b)\right)$; aggregate demand in

[2] A strategy for trader i induces a mapping $s \mapsto F^i_s$, where F^i_s is the cumulative distribution over bids for signal s. The aggregate $F_s(b)$ is obtained by integrating $F^i_s(b)$ over all traders. We describe this aggregation in more detail in Section 5.1 and provide formal details in Appendix A.2.

state $(2,2)$ is $1-\left(\frac{1}{3}F_{H_A}(b)+\frac{1}{6}F_{L_A}(b)+\frac{1}{3}F_{H_B}(b)+\frac{1}{6}F_{L_B}(b)\right)$; and aggregate demand in state $(1,3)$ is $1-\left(\frac{1}{6}F_{H_A}(b)+\frac{1}{3}F_{L_A}(b)+\frac{1}{2}F_{H_B}(b)\right)$. In particular, for any b, the aggregate demand in state $(2,2)$ is a half-half mixture of aggregate demands in states $(3,1)$ and $(1,3)$, because the conditional distribution over signals in state $(2,2)$ is a half-half mixture of the conditional distributions in states $(3,1)$ and $(1,3)$. The market-clearing price in state $(2,2)$ must therefore be between the market-clearing prices in states $(3,1)$ and $(1,3)$.

This example reflects a general property of the auction-mechanism: if the conditional distribution over signals in state ω is between the conditional distributions in states ω' and ω'', then the market-clearing price in state ω is also between the prices in states ω' and ω''. In the current example, this leads to a failure of information aggregation when inputs are complementary because, while the price in state $(2,2)$ must be between the prices in states $(3,1)$ and $(1,3)$, the value in state $(2,2)$ is strictly greater than the value in states $(3,1)$ and $(1,3)$ (see Figure 2b). As a result, for any strategy-profile, the asset must be mis-priced in at least one of these states. However, when traders predict a price that is strictly less than the value, they have an incentive to increase bids; if the price is strictly greater than the value, traders have an incentive to decrease bids. As traders respond to these arbitrage opportunities, competitive forces apply upward pressure on prices in states where the asset is undervalued, and downward pressure on prices in states where the asset is overvalued. This no-arbitrage condition is a general property of equilibrium in the large auction: if prices aggregate information, they must equal values. In the current example, prices cannot equal values, and so equilibrium prices cannot aggregate information.

What happens with asymmetric signals? Finally, consider the case where signals can be asymmetric ($\alpha \neq \frac{1}{2}$). In the symmetric case, complementarities impede information aggregation because the conditional distribution over signals in state $(2,2)$ is between the conditional distributions in states $(3,1)$ and $(1,3)$, but $v(2,2)$ is not between $v(3,1)$ and $v(1,3)$. Asymmetries can resolve this problem. If input A has a much greater impact on the value than input B, then $v(3,1) \geq v(2,2) > v(1,3)$ even when $\rho < 1$. From the previous argument, this condition is clearly necessary for information aggregation, but is it also sufficient? Instead of constructing strategies explicitly, we can appeal to our main result to show the following:[3]

[3]To simplify exposition, we consider $\alpha \geq \frac{1}{2}$, the argument for the opposite case being symmetric.

Corollary 1. *For any $\rho \leq 1$, there exists $\alpha_\rho \in \left[\frac{1}{2}, 1\right)$ such that equilibrium prices can aggregate information if and only if $\alpha \leq \alpha_\rho$. Moreover, α_ρ is continuous and strictly decreasing in ρ,* $\lim_{\rho \to 1} \alpha_\rho = \frac{1}{2}$ *and* $\lim_{\rho \to -\infty} \alpha_\rho = 1$.

Corollary 1 follows from our main result (Theorem 1), which shows the betweenness property is necessary and sufficient for equilibrium prices to aggregate information.

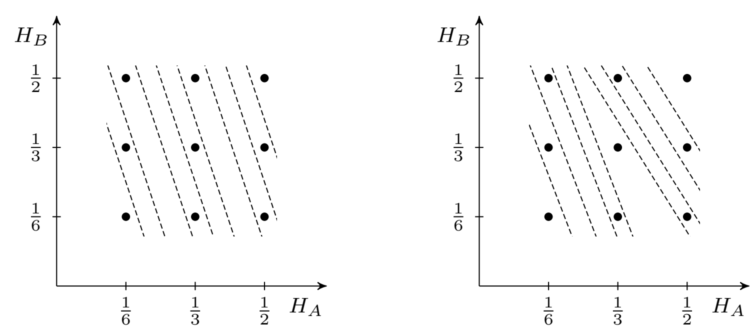
When $\rho = 1$, iso-value lines are linear and it is always possible to find a linear function that is monotone in values. The betweenness property is therefore satisfied.

When $\rho < 1$, $v(3,1) \geq v(2,2)$ is a necessary and sufficient condition for equilibrium prices to aggregate information. This condition is necessary because, with $\alpha > \frac{1}{2}$ and $\rho < 1$, $v(2,2) > v(1,3)$ and when $v(2,2) > v(3,1)$, a high-value state is therefore in the convex hull of two lower-value states, which is inconsistent with betweenness. The condition is also sufficient because it implies the following order over the values:

$$v(3,3)>v(3,2)>\max\{v(2,3),v(3,1)\}\geq v(2,2)\geq \max\{v(2,1),v(1,3)\}>v(1,2)>v(1,1).$$

As Figure 4 illustrates, with this order over values, it is always possible to separate the conditional distributions for each state with hyperplanes, which represent level sets for a quas-linear function $V : \Delta \to \mathbb{R}$ that is monotone in values.

As a result, the condition $v(3,1) \geq v(2,2)$ characterizes a threshold α_ρ such that information aggregation is possible if and only if $\alpha \geq \alpha_\rho$. For small values of α (close to $\frac{1}{2}$), $v(2,2) > v(3,1)$ and so equilibrium prices cannot aggregate information, while for larger values of α (close to 1), $v(3,1) > v(2,2)$ and equilibrium prices can aggregate information.



(a) $\rho = -1$ and $\alpha \in [0.8, 1)$ (b) $\rho = -1$ and $\alpha \in [0.75, 0.8)$

Figure 4: CES value function with asymmetric inputs.

3 Related literature

Our work primarily contributes to a literature using common-value auctions to study the information revealed by prices in a competitive market.[4]

In a seminal contribution, Wilson (1977) shows how equilibrium prices in a single-unit auction can converge in probability to values as the population grows. Milgrom (1979) provides the first characterization of environments that permit aggregation, and Milgrom (1981) extends the analysis to general Vickrey auctions. To overcome the winner's curse—which intensifies when assets become scarce—aggregation requires that the winning bidder's signal is arbitrarily precise. This imposes a strong restriction on information. Pesendorfer and Swinkels (1997) therefore consider auctions where both the number of traders n and the number of assets g increases. When signals satisfy the MLRP, they show that equilibrium prices converge in probability to values if and only if $g \to \infty$ and $(n-g) \to \infty$, which ensures that a loser's curse offsets the winner's curse. Kremer (2002) extends the analysis to characterize the asymptotic distribution of prices for various auction formats in a unified framework. To address limitations of a market with exogenous supply, Reny and Perry (2006) consider a double-sided auction. In an environment with affiliated common and private-values (which implies the MLRP), they show that when the population is sufficiently large there is a monotone equilibrium where prices are close to a fully-revealing REE.

The auction approach has important advantages over the REE literature inspired by Grossman and Stiglitz (1976) and Radner (1979). In a REE, individual trades are generally not measurable with respect to private signals, and this obscures the connection between individual behavior and market outcomes (see, e.g., Hellwig, 1980; Milgrom, 1981). By contrast, in an auction, traders condition bids only on private signals, imposing a natural restriction on information transmission.

However, the auction approach also has limitations. In a finite auction, each trader has market power. When traders internalize market power, they strategically adjust bids so as not to reveal private information, thereby distorting the information conveyed by equilibrium prices. Moreover, these distortions do not vanish as the

[4] A parallel literature studies information aggregation in common-value elections (Condorcet, 1785; Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1997). The closest work in this literature to ours is Barelli, Bhattacharaya, and Siga (2019), who analyze a multi-candidate election with private information and, employing a similar geometric approach to ours, show when a voting strategy can aggregate information.

population grows because traders condition on the pivotal event where they determine the price (Milgrom and Weber, 1982). While the probability of price impact vanishes, the behavior of individual traders therefore does not converge to the price-taking behavior of a competitive market. To overcome these challenges, prior work on auction has imposed the MLRP to construct an equilibrium. But with a continuum of states, the MLRP is non-generic (see, e.g., de Castro, 2010), and also has rapidly vanishing measure in environments with finite states (see Section 6).

Our approach combines insights from both literatures. In particular, we study an auction where traders can condition bids only on private signals, but where a large population implies that each trader is a price-taker. As in a REE, competition therefore manifests solely in the arbitrage behavior of traders. This reflects the important economic idea that, in a large anonymous market, traders believe they cannot impact prices, and the competition for resources—rather than market power—drives individual and aggregate behavior. In such a market, we show that equilibrium prices can aggregate information even in complex, multidimensional environments where signals have no meaningful order, and provide conditions for generic existence of fully-revealing equilibrium prices in an auction-mechanism.[5]

Moreover, in Mihm and Siga (2020b), we show that the restrictions we identify on the market trading mechanism apply also to approximate (and therefore exact) equilibria in finite auctions. Regardless whether or not market power distorts behavior, the trading mechanism simply cannot aggregate information when the betweenness property is not satisfied. In this regard, we also contribute to a literature on failures of information aggregation. For instance, costly information acquisition (Jackson, 2003), uncertainty about the number of bidders (Harstad, Pekeč, and Tsetlin, 2008), costly bidder solicitation (Lauermann and Wolinsky, 2017), state-dependent actions (Atakan and Ekmekci, 2014), or decentralized bilateral trading (Wolinsky, 1990), have all been shown to impede information aggregation even in environments where the MLRP is satisfied. We show that, even without these additional features, there are limitations of the auction price-mechanism.

There are also alternative approaches to provide microfoundations for REE. A literature following Kyle (1985) studies markets with strategic traders who receive private information, non-strategic noise traders who supply liquidity, and a market

[5]Serrano-Padial (2012) and Bodoh-Creed (2013) also study auctions with an infinite population of traders but focus exclusively on environments where signals satisfy the MLRP.

maker who determines prices. Trading is dynamic and information revelation occurs over time. The information aggregation process is therefore quite different from the auction approach because there is feedback from prices. There are also significant differences in the trading mechanism because all orders are executed; in an auction, bids are conditional orders that are executed only when the price is below a threshold. Moreover, strong information assumptions are needed to solve for an equilibrium: random variables are conditionally *i.i.d* and jointly normal, which implies the MRLP.

In an important recent contribution, Lambert, Ostrovsky, and Panov (2018) consider a single-period version of the Kyle model, maintaining joint-normality but relaxing the *i.i.d.* conditions. Their model admits a unique linear equilibrium where prices aggregate information asymptotically if and only if noise trade is positively correlated with values. There are significant differences with our work: (i) our trading mechanism is very different, (ii) our model does not have noise traders, (iii) our large population implies that individual traders have no price impact, and (iv) our environment has finite states and signals, but we impose no distributional assumption on the the joint-probability over states and signals.

There is also a literature on information aggregation where traders submit monotone demand (or supply) schedules (Kyle, 1989; Vives, 2011, 2014).[6] The closest paper in this literature to ours is Palfrey (1985), who studies Cournot competition in an environment with finite states and signals, and an exogenous demand for goods. He does not provide a complete characterization of the environments where information aggregates, but shows that a necessary condition (which is almost sufficient) is that the matrix of conditional distributions has full-rank. In a market where traders do not have price impact, we show that this condition is sufficient for information aggregation because it implies a linear property, which implies the betweenness property. However, the full-rank condition is not necessary for aggregation because (i) the full-rank condition is sufficient but not necessary for the linear property, and (ii) the linear property is sufficient but not necessary for the betweenness property.

[6] Vives (2014) also considers a market with infinitely many traders. To address the Grossman-Stigliz critique, he shows that a fully revealing REE can be implemented as a Bayes-Nash equilibrium when traders acquire costly information about both a private and common value component of the asset. In his model, random variables are jointly normal and signals therefore satisfy the MLRP.

4 Model

We study a uniform-price auction with a large population of traders and an exogenous supply of assets. The common value of a unit of the asset depends on an unknown state, and traders receive private signals that are *i.i.d.* conditional on the state. In this market, we are interested in the information conveyed by equilibrium prices.

4.1 The environment

The environment has a finite set of states $\Omega=\{\omega_1, ..., \omega_M\}$ and signals $S=\{s_1, ..., s_K\}$, with a probability distribution P on $\Omega \times S$. We denote the set of probability distributions over signals by $\Delta = \{x \in \mathbb{R}^K : \mathbf{0} \leq x, x \cdot \mathbf{e} = 1\}$, where $\mathbf{0} = (0, ..., 0)$ is the origin and $\mathbf{e} = (1, ..., 1)$ is the vector of ones. We denote by δ_k the Dirac distribution with probability 1 on signal s_k. In state ω, a unit of asset has value $v(\omega) \geq 0$ and the conditional distribution over signals is $P_\omega \in \Delta$. We assume that each state occurs with strictly positive probability, and that different states generate different conditional distributions. The key primitives of the environment are the value function $v : \Omega \to \mathbb{R}_+$ and information structure $\{P_\omega : \omega \in \Omega\}$.

Information aggregation requires a relationship between the value function and the information structure, which can be described in terms of a quasi-linear function.

Definition 1. A function $V : \Delta \to \mathbb{R}$ is *quasi-linear* if it satisfies two conditions.

(i) *Lower semicontinuity:* all lower contour sets $\{y : V(y) \leq V(x)\}$ are closed.

(ii) *Betweenness:* $V(x) \geq V(y)$ implies $V(x) \geq V(\theta x + (1-\theta)y) \geq V(y)$.

Lower semicontinuity is a technical condition, which describes how function values can be approximated (from below, but not necessarily from above). Betweenness is the substantive condition. It implies that lower contour sets, level sets and upper contour sets are all convex. In particular, the boundaries of these sets can be represented by hyperplanes. An important special case are linear functions, where $V(x) = \alpha \cdot x$ for some $\alpha \in \mathbb{R}^K$. While linear functions are those (continuous) functions that are both concave and convex, quasi-linear functions are those (lower semicontinuous) functions that are both quasi-concave and quasi-convex.

We use quasi-linear functions to define a property of the environment, which establishes an ordinal relationship between values and conditional distributions.

Definition 2. An environment satisfies the *betweenness (resp. linear) property* if there is a quasi-linear (resp. linear) function $V : \Delta \to \mathbb{R}$ that is monotone in values: $v(\omega) > v(\omega')$ implies $V(P_\omega) > V(P_{\omega'})$.

Since a linear function is quasi-linear, the linear property implies the betweenness property. Figure 1 in the introduction illustrates environments where (a) the linear property is satisfied, (b) the linear property is not satisfied but the betweenness property is satisfied, and (c) the betweenness property is not satisfied. In general, a quasi-linear function can have infinitely many level sets, which can cover the whole simplex. Since our environments have finite states and signals, it is sufficient for us to depict a finite number of the level sets. Crucial for the betweenness property is that (i) the level sets are linear, (ii) the upper contour sets are nested on the simplex, and (iii) higher values correspond to conditional distributions on higher level sets. The linear property requires, in addition, that the level sets are parallel.

There is a direct connection between the level sets of a quasi-linear function and the iso-price lines generated by bidding strategies in an auction. We discuss this connection in detail after describing the market and stating our main result.

4.2 The market

There is an infinite population of traders $\mathcal{I}$ with mass 1, and a mass $\kappa \in (0, 1)$ of assets. Each trader has negligible mass, and so cannot impact prices.

Nature chooses a state ω according to the marginal distribution on Ω. Traders do not observe the state, but receive a private signal drawn independently from the conditional distribution P_ω. After receiving their signals, each trader submits a sealed bid from a compact interval $B \equiv [0, \bar{b}]$, which contains $v(\Omega)$. A bid represents the maximum price at which a trader is willing to purchase a unit of asset.

Auction mechanism: The auction format provides an explicit protocol for the price formation process. Given a bid-profile $a : \mathcal{I} \to B$, where $a(i)$ represents trader i's bid, the auctioneer determines a price and an allocation of assets.[7] The price $p(a)$ is the lowest bid at which the mass of traders willing to trade exceeds the supply of assets, and all trade occurs at this price. A trader receives a unit of the asset if her

[7]The set of bid-profiles $\mathcal{A} = \{a : \mathcal{I} \to B\}$ is endowed with the σ-algebra $\mathbb{A}$ generated by cylinder sets of the form $\{a : a(i) = b\}$ for some $i \in \mathcal{I}$ and $b \in B$.

own bid is strictly above the price and does not trade if her bid is strictly below the price. To clear the market, the auctioneer uniformly randomizes over bids equal to the price. This allocation rule ensures that the market clears, treats market participants symmetrically, and guarantees that (i) no trader wins the auction with a bid strictly below the price and (ii) no trader loses the auction with a bid strictly above the price. In state ω, the payoff for a trader is $v(\omega) - p(a)$ if she trades, and 0 otherwise.

Strategies and equilibrium: A strategy-profile $\sigma : \mathcal{I} \times S \to \mathcal{B}$ is a mapping from types to Borel distributions over bids, where $\sigma(i, s)$ is the (mixed) bidding strategy for trader i when she receive signal s. A strategy-profile σ and conditional distribution P_ω generate a probability measure P_ω^σ over bid-profiles in state ω. The expected payoff for type (i, s) is $\Pi_i(\sigma|s) \equiv \sum_\omega \Pi_i(\sigma|\omega)P_s(\omega)$, where $P_s(\omega)$ is the probability of state ω conditional on signal s, $\Pi_i(\sigma|\omega) \equiv \int_{\mathcal{A}} \pi_i(a|\omega)dP_\omega^\sigma$ is the expected payoff conditional on state ω, and $\pi_i(a|\omega)$ is trader i's payoff in state ω for the bid-profile a. A strategy-profile is a Bayes-Nash equilibrium (henceforth, *equilibrium*) if each type maximizes their expected payoff given the strategy of other types. Our result also holds if equilibrium requires only that *almost all* types are playing a best-response.

Aggregate demand: Following common practice in the literature, we appeal to the LLN to describe the aggregate demand for assets.[8] For a strategy-profile σ, let $F^{\sigma(i,s)}$ denote the cumulative distribution over bids by trader i when she receives signal s. Since traders randomize independently, the LLN implies that $F_s^\sigma(b) \equiv \int_{\mathcal{I}} F^{\sigma(i,s)}(b)\lambda(di)$ is the mass of traders who bid less than b when they receive signal s. The aggregate bid distribution is therefore described by the vector of cumulative distribution functions $F^\sigma \equiv \left(F_{s_1}^\sigma, ..., F_{s_K}^\sigma\right)$.

In state ω, traders independently draw private signal from the conditional distribution P_ω. As a result, the LLN implies that $P_\omega(s)$ is the mass of traders who receive signal s, and the mass of traders who submit a bid less than or equal to b is $F_\omega^\sigma(b) \equiv F^\sigma(b) \cdot P_\omega$. The decumulative distribution function $1 - F_\omega^\sigma$ can then be interpreted as the aggregate demand function in state ω because, for any $b \in B$, $1 - F_\omega^\sigma(b)$ is the mass of traders willing to buy at price b.

[8]To focus on the economic intuition of our main result, we appeal informally to the LLN. In Mihm and Siga (2020b), we follow an approach by Al-Najjar (2008) to provide an explicit model of the large population where we derive, rather than assume, the LLN and show that the aggregate demand and pricing-functions for the auction can be established formally.

Market price: Aggregate demand is equal to supply when $1 - F^\sigma_\omega(b) = \kappa$. Since F^σ_ω may not be continuous or strictly increasing, there is not always a unique bid that satisfies this market-clearing equation. In such cases, the market price is equal to the $(1-\kappa)$-quantile of F^σ_ω, denoted $\mathcal{Q}^\sigma_\omega(1-\kappa)$ (i.e., the price is equal to the lowest bid b^* such that there is excess supply ($\kappa \geq 1 - F^\sigma_\omega(b)$) for all higher bids $b \geq b^*$). As such, we appeal to the LLN to ensure that, for any strategy-profile σ, there is a price-function $p_\sigma : \Omega \to B$ such that, in state ω, the price is equal to $p_\sigma(\omega)$ almost surely (i.e., for every state ω there is a measurable subset of bid-profiles A such that $P^\sigma_\omega(A) = 1$ and $p(a) = p_\sigma(\omega)$ for all $a \in A$.)

5 Main result

We are interested in strategy-profiles where prices convey the same information about values as would be obtained with public signals. By the LLN, the proportion of traders who receive signal s in state ω is almost surely equal to $P_\omega(s)$. Public signals therefore reveal the value almost surely, and a strategy-profile conveys the same information if only if there is a one-to-one mapping between values and prices.

Definition 3. Strategy-profile σ *aggregates information* if $v(\omega) \neq v(\omega')$ implies $p_\sigma(\omega) \neq p_\sigma(\omega')$.

It is always possible to construct a strategy-profile that aggregates information. However, we are interested in strategies where traders respond to incentives generated by the competition for assets. While an individual trader has negligible impact on the price, she can affect her allocation and thereby influence her expected payoff. In particular, traders will try to exploit arbitrage opportunities based on their predictions about prices and values. Accordingly, the aggregate supply and demand for assets depends on the incentives of the traders, and our main result characterizes when *equilibrium* prices aggregate information.

Theorem 1. *There is an equilibrium strategy-profile that aggregates information if and only if the betweenness property is satisfied.*

By connecting the aggregation problem directly with primitives, Theorem 1 distinguishes between two types of environments. When the betweenness property is satisfied, equilibrium prices can aggregate all private information in the market. This highlights the potential of the market: even if individual traders are poorly informed, competitive forces can coordinate individual behavior so that prices are perfectly informative. On the other hand, when the betweenness property is not satisfied, information aggregation necessarily fails. This highlights the limitations of the market: even if the population as a whole is perfectly informed, the market cannot guide traders' to reveal their collective information.

5.1 Proof sketch

We provide a sketch of the proof, in order to show where prices originate, and why the betweenness property is necessary and sufficient for information aggregation.

Properties of market-clearing prices: We first identify properties of market-clearing prices, which apply to any strategy-profile σ.

To establish all pricing implications of the auction-mechanism, we need to first extend the price function $p_\sigma : \Omega \to B$ to the whole simplex. This extension is possible because traders condition bids only on private signals, and strategies are therefore independent of value. As a result, a strategy-profile σ imputes a price for *any* distribution over signals $x \in \Delta$, even if no state generates distribution x. In a large auction, the extension is especially tractable because aggregate demand is additively separable in a component F^σ, which depends only on bidding behaviour. We can therefore impute a price for $x \in \Delta$ by letting $\tilde{p}_\sigma(x)$ be the $(1-\kappa)$-quantile of $F^\sigma_x \equiv F^\sigma \cdot x$. The *extended price-function* $\tilde{p}_\sigma : \Delta \to B$ thereby assigns a price to every distribution over signals, and satisfies $\tilde{p}_\sigma(P_\omega) = p_\sigma(\omega)$ for every state ω.

The key restriction imposed by the auction-mechanism on prices is that the extended price function $\tilde{p}_\sigma$ is quasi-linear. To prove this, we need to show that $\tilde{p}_\sigma$ satisfies lower semi-continuity and betweenness.

(i) Lower semicontinuity: Suppose $\tilde{p}_\sigma(x) > p$. Since $\tilde{p}_\sigma$ is the $(1-\kappa)$-quantile of F^σ_x, it follows that $F^\sigma(p) \cdot x < 1-\kappa$. Therefore, $F^\sigma(p) \cdot y < 1-\kappa$ for all y in an open neighborhood around x, and so $\tilde{p}_\sigma(y) > p$. Since the strict upper contour sets of $\tilde{p}_\sigma$ are open, the lower contour sets are closed, and so $\tilde{p}_\sigma$ satisfies lower semicontinuity.

(ii) Betweenness: Suppose $\tilde{p}_\sigma(x) \geq \tilde{p}_\sigma(y)$, and let $z = \theta x + (1-\theta)y$ for some $\theta \in [0,1]$. For all $b < \tilde{p}_\sigma(y)$, $F^\sigma(b) \cdot x < 1-\kappa$ and $F^\sigma(b) \cdot y < 1-\kappa$, and therefore $F^\sigma(b) \cdot z < 1-\kappa$. On the other hand, for all $b \geq \tilde{p}_\sigma(x)$, $F^\sigma(b) \cdot x \geq 1-\kappa$ and

$F^\sigma(b) \cdot y \geq 1 - \kappa$, and therefore $F^\sigma(b) \cdot z \geq 1 - \kappa$. As a result, the $(1-\kappa)$-quantile of F^σ_z is between $\tilde{p}_\sigma(y)$ and $\tilde{p}_\sigma(x)$, and so $\tilde{p}_\sigma$ satisfies betweenness.

Lower semicontinuity implies that auction prices can be approximate from below. Betweenness is the more substantive restriction of the auction-mechanism. Since prices depend only on a single quantile of the aggregate bid distribution, average conditional distributions must generate average prices for any strategy-profile.

Converse: We show that there are no other general properties of market-clearing prices by establishing a converse: for *any* quasi-linear function $V : \Delta \to B$, there is a strategy-profile σ so that $\tilde{p}_\sigma(x) = V(x)$ for all $x \in \Delta$.

We first provide intuition for the proof by considering the special case where V is binary: $V(x) > V(y)$ for some $x, y \in \Delta$, and $V(z) \in \{V(x), V(y)\}$ for all $z \in \Delta$. In that case, let $U_y = \{z : V(z) > V(y)\}$ and $L_y = \{z : V(y) \geq V(z)\}$. Both sets are non-empty because $x \in U_y$ and $y \in L_y$. Since V is lower semicontinuous, L_y is closed and U_y is open. Betweenness implies that both sets are convex. Therefore, by the Hyperplane Separation Theorem, there are non-zero $\alpha \in \mathbb{R}^K$ and constant $c \in \mathbb{R}$ so that $\alpha \cdot z \leq c$ for all $z \in L_y$ and $\alpha \cdot z > c$ for all $z \in U_y$.

The following observation is central to the argument. Since the unit simplex Δ is a lower-dimensional subspace of $\mathbb{R}^K$, the separating hyperplane is not unique: there are many hyperplanes in $\mathbb{R}^K$ that have the same intersection with the unit simplex, and therefore achieve the desired separation between L_y and U_y. We exploit this degree of freedom to choose a separating hyperplane that has the same intersection with the unit simplex as the hyperplane $H(\alpha, c)$, but where each component of the norm is between 0 and 1, and the constant is equal to $1 - \kappa$. We can then interpret components of the norm as bidding probabilities, and relate the separation to market-clearing conditions.

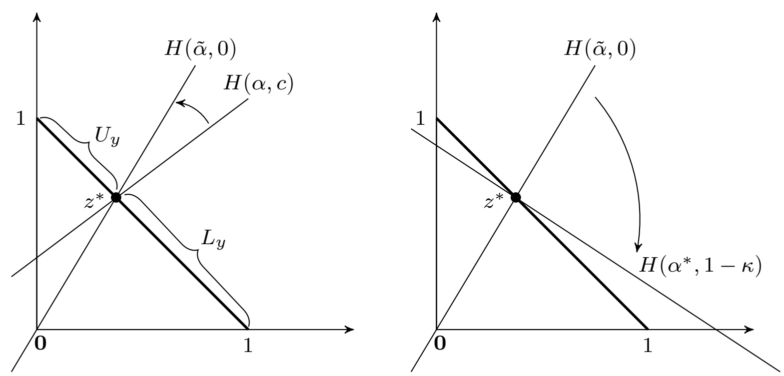
To construct the desired hyperplane, we proceed in two steps. For the first step, let $\tilde{\alpha} = c\mathbf{e} - \alpha$ and consider the hyperplane $H(\tilde{\alpha}, 0)$, which has norm $\tilde{\alpha}$ and constant 0. This hyperplane also separates L_y from U_y because, for all $z \in \Delta$,

$$\tilde{\alpha} \cdot z = \alpha \cdot z - c(\mathbf{e} \cdot z) = \alpha \cdot z - c \leq 0 \quad \Leftrightarrow \quad \alpha \cdot z \leq c.$$

For the second step, let $\phi = \max_k |\tilde{\alpha}(k)|$, which is strictly positive because $\tilde{\alpha} \cdot x < 0$, and let $\tilde{\phi} = \max\{\phi/\kappa, \phi/(1-\kappa)\} > 0$. Now let $\alpha^* = \tilde{\phi}^{-1}\tilde{\alpha} + (1-\kappa)\mathbf{e}$ and consider the hyperplane $H(\alpha^*, 1-\kappa)$, which has norm α^* and constant $1-\kappa$. This hyperplane also separates L_y from U_y because, for all $z \in \Delta$,

$$\alpha^* \cdot z = \frac{1}{\tilde{\phi}}\tilde{\alpha} \cdot z + (1-\kappa)\mathbf{e} \cdot z = \frac{1}{\tilde{\phi}}\tilde{\alpha} \cdot z + 1 - \kappa \leq 1 - \kappa \quad \Leftrightarrow \quad \tilde{\alpha} \cdot z \leq 0$$

Moreover, $\mathbf{0} \leq \alpha^* \leq \mathbf{e}$ because $-(1-\kappa) \leq \frac{1}{\tilde{\phi}}\tilde{\alpha}(k) \leq \kappa$ for all $k = 1, ..., K$. Figure 5 provides an illustration of these transformations for an environment with two signals, where the boundary between L_y and U_y can be represented by a single point z^*.



(a) First rotation (b) Second rotation

Figure 5: Separating hyperplanes.

In panel (a), hyperplane $H(\tilde{\alpha}, 0)$ is a rotation of $H(\alpha, 0)$ around the point z^*, which still separate L_y from U_y but now passes through the origin. In panel (b), hyperplane $H(\alpha^*, 1-\kappa)$ is a rotation of $H(\tilde{\alpha}, 0)$ around the point z^*, which intercepts the vertical axes at $\frac{(1-\kappa)\tilde{\phi}}{\tilde{\alpha}(2)+(1-\kappa)\tilde{\phi}}$ and the horizontal axes at $\frac{(1-\kappa)\tilde{\phi}}{\tilde{\alpha}(1)+(1-\kappa)\tilde{\phi}}$. This hyperplane still separates L_y from U_y, but each component of the norm is between 0 and 1 and constant is $1-\kappa$.

Given a hyperplane with the desired properties, we can now construct a strategy-profile σ with $\tilde{p}_\sigma \equiv V$. When trader i receives signal s_k, she bids $V(y)$ with probability $\alpha^*(k)$ and $V(x)$ with probability $1-\alpha^*(k)$. If all types follow this strategy, then for all $z \in \Delta$,

$$F_z^\sigma(b) = \begin{cases} 0 & \text{if } b < V(y) \\ \alpha^* \cdot z & \text{if } b \in [V(y), V(x)) \\ 1 & \text{if } b \geq V(x) \end{cases}.$$

If $z \in L_y$, then $F_z^\sigma(b) = 0 < 1-\kappa$ for $b < V(y)$ and $F_z^\sigma(b) \geq 1-\kappa$ for $b \geq V(y)$, and so the $\tilde{p}_\sigma(z) = V(y) = V(z)$. On the other hand, if $z \in U_y$, then $F_z^\sigma(b) < 1-\kappa$ for $b < V(x)$ and $F_z^\sigma(b) = 1$ for $b \geq V(x)$, and so $\tilde{p}_\sigma(z) = V(x) = V(z)$. As a result, the strategy-profile σ induces an extended price-function with $\tilde{p}_\sigma(z) \equiv V(z)$.

Proposition 6 in Appendix A.2 generalizes this argument to any quasi-linear function $V : \Delta \to B$, thereby showing that lower semicontinuity and betweenness characterize the restrictions that the auction-mechanism imposes on prices. The geometric intuition behind the general argument is similar to the binary case. In particular, betweenness implies that all lower and upper contour sets are convex, and can therefore be separated by hyperplanes. As in the binary case, one can rotate each separating hyperplane so that each component of the norm is between 0 and 1, and the constants is $1 - \kappa$. The additional step, established in Lemma 4 in the Appendix, is to show that hyperplanes can be rotated so that the norms are totally ordered: if α_x is the norm of the hyperplane separating U_x from L_x, then $\alpha_x \geq \alpha_y$ if and only if $V(x) \geq V(y)$. This order property of the separating hyperplanes allows us to interpret the norms as *cumulative* probabilities in the construction of a strategy-profile.

No-arbitrage property: In an equilibrium, competitive forces impose additional discipline on market prices: if the strategy-profile σ aggregates information, then σ is an equilibrium if and only if $p_\sigma \equiv v$. We provide the intuition for this equilibrium property here, and defer a formal proof to Appendix A.2.

"If" follows directly from the competition between a large population of traders. In particular, since individual traders cannot impact prices, when $p_\sigma(\omega) = v(\omega)$ in every state the expected payoff for any deviation is zero.

For the converse, suppose that the strategy-profile σ aggregates information but there is some state ω where the asset is underpriced, i.e., $p_\sigma(\omega) < v(\omega)$. Since σ aggregates information, there is an interval around the market-clearing price so that $p_\sigma(\omega') \notin [p_\sigma(\omega) - \varepsilon, p_\sigma(\omega) + \varepsilon]$ for all states where $v(\omega') \neq v(\omega)$. A strictly positive mass of traders must submit bids in the interval $(p_\sigma(\omega) - \varepsilon, p_\sigma(\omega))$, otherwise aggregate demand would equal supply already at the lower price $p_\sigma(\omega) - \varepsilon$. These traders do not win the auction in state ω even though it would be profitable to do so. Moreover, the traders can deviate from σ by holding fixed their distribution over bids outside the interval $[p_\sigma(\omega) - \varepsilon, p_\sigma(\omega) + \varepsilon]$, but moving all probability mass from $[p_\sigma(\omega) - \varepsilon, p_\sigma(\omega))$ to $(p_\sigma(\omega), p_\sigma(\omega) + \varepsilon]$. This deviation does not affect the probability of trading in states where $v(\omega') \neq v(\omega)$, but strictly increases the probability of winning the auction in state ω. Since the asset is underpriced in state ω, this deviation is strictly profitable, and σ is not an equilibrium. If the asset is overpriced in any state, there must be another state where the asset is underpriced, otherwise always bidding 0 is a strictly profitable deviation. No-arbitrage conditions therefore imply that, if equilibrium prices aggregate information, they must equal values.

Information aggregation: Finally, we combine the quasi-linearity of prices, which is satisfied for any strategy-profile, with the no-arbitrage property of prices, which is satisfied in an equilibrium.

First, suppose σ is an equilibrium that aggregates information. The extended price function $\tilde{p}_\sigma$ is quasi-linear and, by the no-arbitrage property, is also monotone in values. As a result, the betweenness property is satisfied.

Conversely, suppose there is a quasi-linear function $V : \Delta \to \mathbb{R}$ that is monotone in values. We can always normalize V to ensure that it maps to B, and is still quasi-linear and monotone in values.[9] As a result, there is a strategy-profile σ so that $\tilde{p}_\sigma(x) = V(x)$ for all $x \in \Delta$.

The strategy-profile σ may not be an equilibrium, but we can re-scale bids to ensure that prices equal values. We demonstrate the re-scaling when $v(\omega_m) < v(\omega_{m+1})$ for $m = 1, ..., M-1$, and defer the case where v is not injective to Appendix A.4.

Consider the strategy-profile $\hat{\sigma}$ where type (i, s) submits bid $v(\omega_1)$ with probability $F_s^\sigma(p_\sigma(\omega_1))$, submits bid $v(\omega_m)$ with probability $F_s^\sigma(p_\sigma(\omega_m)) - F_s^\sigma(p_\sigma(\omega_{m-1}))$ for $m = 2, ...M-1$, and submits bid $v(\omega_M)$ with probability $1 - F_s^\sigma(p_\sigma(\omega_{M-1}))$. This is a well-defined probability distribution on B because $\tilde{p}_\sigma$ is monotone in values, and so $F_s^\sigma(p_\sigma(\omega_m)) - F_s^\sigma(p_\sigma(\omega_{m-1})) \geq 0$ for $m = 2, .., M-1$. Moreover, if all types follow this strategy, then in state ω the LLN implies that

$$F_\omega^{\hat{\sigma}}(b) = \begin{cases} 0 & \text{if } b < v(\omega_1) \\ F_\omega^\sigma(p_\sigma(\omega_m)) & \text{if } b \in [v(\omega_m), v(\omega_{m+1})) \text{ and } m = 1, ..., M-1 \\ 1 & \text{if } b \geq v(\omega_M) \end{cases} .$$

Since $p_\sigma(\omega)$ is the $(1-\kappa)$-quantile of F_ω^σ, it follows that $v(\omega_m)$ is the $(1-\kappa)$-quantile of $F_{\omega_m}^{\hat{\sigma}}$ for $m = 1, ..., M$. As a result, $p_{\hat{\sigma}}(\omega) = v(\omega)$ for every state ω, and the strategy-profile $\hat{\sigma}$ is therefore an equilibrium that aggregates information.

[9]Since $\Delta = co(\{\delta_k : k = 1, ..., K\})$, betweenness implies that $\min_k V(\delta_k) \leq V(x) \leq \max_k V(\delta_k)$ for all $x \in \Delta$. Now let $\phi = |\max_k V(\delta_k) - \min_k V(\delta_k)|$. If $\phi = 0$, let $\tilde{V} = V - \min_k V(\delta_k)$; if $\phi > 0$, let $\tilde{V} = \frac{\bar{b}}{\phi}(V - \min_k V(\delta_k))$. Then $\tilde{V} : \Delta \to B$ and, since $\tilde{V}$ is a strictly positive affine transformation of V, it is quasi-linear and monotone in values.

5.2 Discussion

To develop additional insights on the characterization result, we illustrate the connection between prices and quasi-linear functions geometrically, provide some examples to indicate when information aggregation fails, and discuss how the betweenness property is related to the MLRP.

Mapping strategies to iso-price lines: Consider a strategy-profile σ where F_s^σ is continuous and strictly increasing for every signal s. In that case, the market-clearing price in state ω is the unique solution to $(\mathbf{e} - F^\sigma(b)) \cdot P_\omega = \kappa$. For any bid b, we can interpret the vector $\mathbf{e} - F^\sigma(b)$ as the norm of a hyperplane $H(\mathbf{e} - F^\sigma(b), \kappa)$, so that the market-clearing price in state ω is the unique bid b^* such that $P_\omega \in H(\mathbf{e} - F^\sigma(b^*), \kappa)$. Moreover, for any distribution $x \in H(\mathbf{e} - F^\sigma(b^*), \kappa)$, $\tilde{p}_\sigma(x) = b^*$, and so $H(\mathbf{e} - F^\sigma(b^*), \kappa)$ can be interpreted as an iso-price line induced by the strategy-profile σ. Likewise, we can construct iso-price lines for every distribution in Δ. These iso-price lines must be nested: higher iso-price lines are above lower iso-price lines. The market-clearing conditions for strategy-profile σ can therefore be represented by a collection of nested, linear iso-price lines, which is the defining characteristic of a continuous quasi-linear function (see Figure 6).

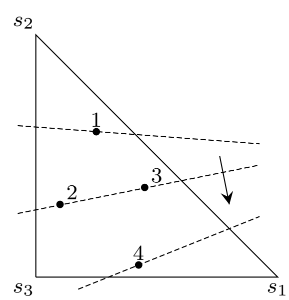


Figure 6: Iso-price lines.

Strategy-profile σ generates linear nested iso-price lines, which therefore represent level sets of a quasi-linear function. Moreover, when σ is an equilibrium that aggregates information, the quasi-linear function is monotone in values.

Failures of information aggregation: To illustrate why the betweenness property is necessary for information aggregation, consider three examples based on Figure 7.

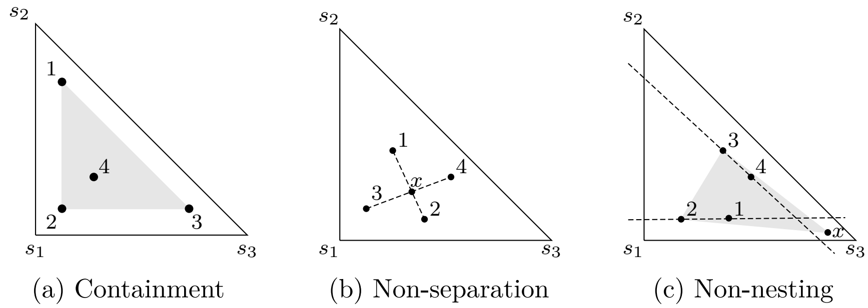


Figure 7: Information aggregation failures.

In all three panels, it is not possible to separate low value states from high value states with nested hyperplanes, and so the betweenness property is not satisfied.

Example 1. In panel (a), P_4 is in the convex hull of $\{P_1, P_2, P_3\}$. Therefore, for some $\beta, \theta \in [0,1]$, $P_4 = \beta(\theta P_1 + (1-\theta)P_2) + (1-\beta)P_3$. As a result, for any strategy-profile σ, $\tilde{p}_\sigma(\theta P_1 + (1-\theta)P_2)$ is between $\tilde{p}_\sigma(P_1)$ and $\tilde{p}_\sigma(P_2)$, and $\tilde{p}_\sigma(P_4)$ is between $\tilde{p}_\sigma(\theta P_1 + (1-\theta)P_2)$ and $\tilde{p}_\sigma(P_3)$. The price in the high value state is therefore an average of the prices in the lower value states, and so the asset must be mis-priced in at least one state. In an equilibrium, the asset must be underpriced in one state and overpriced in another, otherwise traders have arbitrage opportunities. Equilibrium prices therefore pool low and high value states, and cannot aggregate information.

Example 2. In panel (b), there is no state that generates the conditional distribution x over signals, but a strategy-profile σ still imputes a price $\tilde{p}_\sigma(x)$. Since $x \in co(\{P_1, P_2\})$, $\tilde{p}_\sigma(x)$ is between $\tilde{p}_\sigma(P_1)$ and $\tilde{p}_\sigma(P_2)$. Since $x \in co(\{P_3, P_4\})$, $\tilde{p}_\sigma(x)$ is between $\tilde{p}_\sigma(P_3)$ and $\tilde{p}_\sigma(P_4)$. Therefore $[p_\sigma(\omega_1), p_\sigma(\omega_2)]$ and $[p_\sigma(\omega_3), p_\sigma(\omega_4)]$ must intersect, which implies that the asset is either overpriced in one of the low value states, or underpriced in one of the high value states. The no-arbitrage condition implies that this mis-pricing is consistent with equilibrium only if prices pool low and high value states.

Example 3. In panel (c), P_1 and P_4 are in the convex hull of $\{P_2, P_3, x\}$. Therefore, for any strategy-profile σ, $\tilde{p}_\sigma(P_1)$ and $\tilde{p}_\sigma(P_4)$ are in $co(\{\tilde{p}_\sigma(P_2), \tilde{p}_\sigma(P_3), \tilde{p}_\sigma(x)\})$. In order to price the asset correctly when the value is 1, $\tilde{p}_\sigma(x)$ must be less than 1, but then the asset is underpriced when the value is 4. Again, this mis-pricing is consistent with equilibrium no-arbitrage conditions only if prices pool low and high value states.

These examples highlight three features of the market, which impede information aggregation when the betweenness property is not satisfied. First, there is the extension property, whereby a strategy-profile has pricing-implications for any distribution over signals, even if it is not part of the environment. Second, there is the betweenness condition, whereby average conditional distributions generate average prices. Finally, there is the no-arbitrage property, whereby equilibrium prices aggregate information only if they equal values. The combination of these features implies that equilibrium prices can aggregate information only if the betweenness property is satisfied.

The discussion in Examples 1–3 is concerned only with necessary conditions for an equilibrium. We do not have general results on the existence and properties of equilibria where information is not aggregated. The extended price function is quasi-linear for any strategy-profile, and therefore reflects a general limitation of the auction-mechanism. However, the no-arbitrage property is specific to equilibria that aggregate information. Since traders generally have different posterior expectations about values, incentives are complicated when prices pool low and high value states. We do not have a general approach to study equilibria in such cases, and leave this an open question for future work.

Betweenness property and the MLRP: The betweenness property is not only necessary, but also sufficient for information aggregation in a large auction. Below, we compare the betweenness property to the MRLP, which is known to be sufficient for auction prices to aggregate information from the prior literature (see, e.g., Milgrom, 1981; Pesendorfer and Swinkels, 1997; Kremer, 2002):

Definition 4. An environment satisfies the MLRP if there is a weak order $\triangleright$ on signals such that $v(\omega) > v(\omega')$ and $s \triangleright s'$ implies $P_\omega(s)P_{\omega'}(s') \geq P_{\omega'}(s)P_\omega(s')$.

In an environment with two states, the MLRP is always satisfied. With two signals, the betweenness property is satisfied if and only if the MLRP is satisfied. When there are more than two signals and two states, it is straightforward to show that the MLRP implies the betweenness property, and the MLRP is therefore a sufficient condition for information aggregation in a large auction.

Given the prior literature, it is not surprising that the MLRP is sufficient for information aggregation in a large auction, but the betweenness property is a less restrictive condition on primitives. In fact, the MLRP can also be characterized in terms of quasi-linear functions. Given a weak order on signals $\triangleright$, a quasi-linear function $V : \Delta \to \mathbb{R}$ is monotone if $s_k \triangleright s_{k'}$ implies $V(\delta_k) \geq V(\delta_{k'})$. In Mihm and

Siga (2020a), we show that an environment satisfies the MLRP if and only if there is a weak order $\triangleright$ on signals such that $v(\omega) > v(\omega')$ implies $V(P_\omega) > V(P_{\omega'})$ for *every* monotone quasi-linear function. The relation between the MLRP and monotone quasi-linear functions is therefore analogous to the well-known relation between first-order stochastic dominance and monotone linear functions. The betweenness property is less restrictive than the MLRP because (i) it does not require a prior order on signals, and (ii) given an order on signals, the betweenness property requires that conditional distributions are monotone in values for *some* quasi-linear function, while the MLRP requires that conditionals are monotone in values for *all* quasi-linear functions.

5.3 Extensions

We conclude this section by discussing some extensions of the model.

Risk preferences: The assumption that traders are risk neutral simplifies exposition, but is inessential. Suppose each trader $i \in \mathcal{I}$ has a strictly-increasing utility function $u_i : \mathbb{R}_+ \to \mathbb{R}$, where marginal utilities are uniformly bounded away from 0. Given a bid-profile $a : \mathcal{I} \to B$, the payoff for trader i in state ω is then $w_i(a|\omega) u_i (v(\omega) - p(a)) + (1 - w_i(a|\omega))\, u_i(0)$, where $w_i(a|\omega)$ is the probability of winning the auction in state ω with bid-profile a. Our main result extends to this model with heterogeneous risk attitudes because the no-arbitrage property still holds, and quasi-linearity is a general property of the auction price-mechanism.

Asymmetric signals: The sufficiency result can be adapted to environments where traders are not ex-ante exchangeable. Consider a finite partition on the traders $(T_1, ..., T_J)$. Each group has a set of signals S_j and a group-specific information structure $\{P^j_\omega : \omega \in \Omega\}$. If the environment for each group satisfies the betweenness property, then there is an equilibrium that aggregates information. In particular, for each group j one can construct a group-specific strategy-profile so that, in each state ω, the aggregate demand for group j equals $\kappa\lambda(T_j)$ exactly when the price equals the value. Aggregate demand for the population then equals supply at the value.

Finite approximation: In Mihm and Siga (2020b), we show that an equilibrium in the large auction can be approximate by ε-equilibria for an increasing sequence of finite auctions. A sequence of strategy-profiles aggregates information asymptotically if prices become arbitrarily informative about values as the population grows, and approximates equilibrium if profitable deviations vanish (i.e., $\varepsilon \to 0$). In symmetric strategies, we establish a counterpart for Theorem 1: there exists a sequence of strategy-profiles that aggregates information asymptotically and approximates equilibrium if and only if the betweenness property is satisfied.

The finite approximation reflects essentially the same economic intuition as our result for the large market. First, for a sufficiently large population, the LLN disciplines aggregate bidding behavior, so that prices eventually become stable. Second, when prices convey precise information, competition ensure that prices must be close to values to prevent traders from pursuing arbitrage opportunities. In a finite market, traders can also potentially profit by exploiting their impact on prices, but this market power vanishes as the population grows. Finally, the extended price-function converges to a quasi-linear function, and so prices converge in probability to values if and only if the betweenness property is satisfied.

Endogenous supply: Our result can also be adapted to a double-sided auction. Suppose a mass κ of traders are sellers, with a unit endowment of asset, and mass $(1-\kappa)$ are buyers, with unit demand. Sellers submit ask-prices and buyers submit bid-prices to an auction clearing-house, which determines a market-clearing price.

As previously, we can represent the aggregate bidding behavior of the buyers by a cumulative distribution function F_ω^σ, but now the aggregate demand at price p is $(1-\kappa)(1-F_\omega^\sigma(p))$, because only a proportion $(1-\kappa)$ of the traders are buyers. Likewise, the aggregate bidding behavior of the sellers can be represented by a cumulative distribution function G_ω^σ and the aggregate supply is $\kappa G_\omega^\sigma(p)$. Aggregate supply is a cumulative distribution because sellers submit ask-prices not bid-prices. The market-clearing price is the lowest bid b^* so that there is excess supply when $b \geq b^*$, i.e., $\kappa G_\omega^\sigma(b) \geq (1-\kappa)(1-F^\sigma(b))$ or equivalently $\kappa G_\omega^\sigma(b) + (1-\kappa)F_\omega^\sigma(b) \geq (1-\kappa)$. The price is therefore the $(1-\kappa)$-quantile of a cumulative distribution function, which is additively separable in a component $\kappa G^\sigma + (1-\kappa)F^\sigma$ that depends only on bidding behavior. Our arguments for the single-sided auction therefore apply, and equilibrium prices aggregate information if and only if the betweenness property is satisfied.

Private values: We focus on a common-value environment, which is a standard assumption when assets have a fundamental value or when there are secondary markets after uncertainty has been resolved. However, common-values also sidelines issues of the allocation efficiency of market prices (see, e.g., Pesendorfer and Swinkels, 2000). As a result, it would be interesting to extend the analysis to environments with a private-value component in the payoffs.

Our equilibrium construction does not extend to private-value environments, but the betweenness property is a necessary condition for monotone pricing in the auction-mechanism regardless of incentives. For instance, suppose that states induce a conditional distribution over types, which encompass both a private-value component and a signal about a common-value component of payoffs. Then states can still be represented by points in the simplex, a strategy-profile maps types to Borel-distributions over bids, and the price is given by the $(1-\kappa)$-quantile of the aggregate bid distribution. As such, the extended price function is quasi-linear, and is monotone in the common-value component only if the betweenness property is satisfied.

6 Generic information aggregation

Given its fundamental role for information aggregation, we now quantify the likelihood that an environment will satisfy the betweenness property. To simplify exposition, we assume that v is injective and P_ω has full-support for every state. An information structure can be represented by a matrix of dimension $K \times M$, where column m is the conditional distribution over signals in state ω_m. For a given value function, we then quantify environments with the Lebesgue measure μ on $\mathbb{R}^{(K-1)M}$.[10] The following result shows when the betweenness property is generic.

Proposition 1. *The betweenness property has full measure if and only if $K \geq M$.*

The argument for sufficiency is straightforward. For $K \geq M$, the system of linear equations $\alpha \cdot P_{\omega_1}{=}v(\omega_1)$, ..., $\alpha \cdot P_{\omega_K}{=}v(\omega_K)$ has at least as many unknowns as equations and therefore generically has a solution $\alpha^* \in \mathbb{R}^K$. In that case, $V(x) \equiv \alpha^* \cdot x$ defines a linear function on Δ that is monotone in values, and so the betweenness property is satisfied.

[10] In Appendix A.5, we show that the set of information structures, and the subsets satisfying the betweenness property and MLRP, are Lebesgue measurable.

For $M > K$, we show in Appendix A.5 that a strictly positive measure of information structures has a high value state in the convex hull of lower value states, which is inconsistent with the betweenness property (see Figure 8 and Example 1).

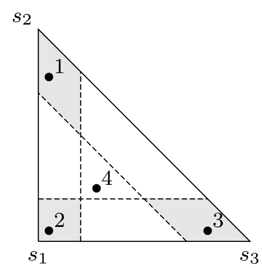


Figure 8: Failure of the betweenness property when $M > K$.

Whenever there is conditional distribution for a low value state in each of the shade regions, and a conditional distribution for a high value state in the middle region, the betweenness property fails. This event has strictly positive Lebesgue measure when $M > K$, and the measure converges to 1 as $M \to \infty$.

Proposition 1 thereby establishes that information aggregation is a generic equilibrium property when there are at least as many signals as states. On the other hand, when $K < M$, there is a strictly positive measure of information structures where the betweenness property fails, and equilibrium prices cannot aggregate information. Moreover, the measure of information structures where the betweenness property is satisfied vanishes as the number of states increases.

Proposition 2. *Fix the number of signals K. Then for any $\varepsilon > 0$, there exists M_ε such the betweenness property has measure less than ε if $M > M_\varepsilon$.*

Proposition 2 extends the necessity argument in Proposition 1 by showing that, if the number of signals remains constant, the likelihood that a high value state is in the convex hull of lower value states converges to one. The likelihood that equilibrium prices can aggregate information therefore depends on the relative number of signals. By way of contrast, the MLRP is generally not satisfied in environments with many states, regardless of the number of signals.

Proposition 3. *For any K, the MLRP has measure bounded above by $\frac{2}{M!}$.*

Proposition 3 implies that, as the number of states increases, the measure of information structures satisfying the MLRP quickly converges to 0, regardless of the number of signals. As a result, there are many environments where the MLRP fails and yet equilibrium prices can aggregate information in a large auction.

7 Multi-input environments

In complex environments, the value of an asset can depend on many uncertain factors, traders generally have specialized knowledge or expertise (e.g., by region or industry), and information can come from a variety of unrelated sources. Our model imposes no *a priori* restrictions on the dimensionality of the states and signals, and the betweenness property can be satisfied even in complex environments, where states and signals are multidimensional.

As an illustration, we consider a class of multidimensional environments that are not reducible to a single dimension.

Definition 5. An environment is a *multi-input environment* with C inputs if (1) $\Omega = \Omega_1 \times ... \times \Omega_C$ and $S = S_1 \cup ... \cup S_C$; (2i) $P(\omega) = \prod_{c=1}^{C} P(\omega_c)$, (2ii) $P(s \in S_c|\omega) = P(s \in S_c)$, and (2iii) $P(s_c|\omega) = P(s_c|\omega_c)$; and (3) $v(\omega) = \psi(\phi_1(\omega_1), ..., \phi_C(\omega_C))$ for a quasi-linear function $\psi : \mathbb{R}^C \to B$ and injective functions $\phi_c : \Omega_c \to \mathbb{R}$.

Multi-input environments are a special case of the environments we have considered thus far. By condition (1), states are multidimensional with one dimension for each input, and each input has a set of signals. By condition (2i), inputs are independent. By condition (2ii), the probability of receiving a signal for input c is independent of the state. By condition (2iii), a signal on input c depends only on the c-th input. Finally, (3) imposes a separability condition on the value function, which aggregates inputs. Multi-input environment therefore provides a stylized model of a market where values depend on multiple sources of uncertainty, but traders receive noisy information about only some dimensions. A multi-input environment is *non-trivial* if $|\Omega_c| > 1$ for at least two inputs. For instance, the example in Section 2 is a non-trivial multi-input environment.[11]

A multi-input environment can always be mapped into an environment where states are unidimensional, but the assumption that signals are informative about only one input is a significant restriction. In particular, the MLRP is never satisfied.

Proposition 4. *A non-trivial multi-input environment does not satisfy the MLRP.*

[11] Conditions (1) and (2) are clearly satisfied for the example in Section 2. For $\rho \neq 0$, let $\phi_A(a) = \alpha a^\rho$, $\phi_B(b) = (1-\alpha)b^\rho$, and $\psi(c,d) = (c+d)^{1/\rho}$; for $\rho = 0$, let $\phi_A(a) = ln(a^\alpha)$, $\phi_B(b) = ln(b^{1-\alpha})$, and $\psi(c,d) = exp(c+d)$. Then ϕ_A and ϕ_B are injective, ψ is quasi-linear, and $v(\omega_A, \omega_B) = \psi(\phi_A(\omega_A), \phi_B(\omega_B))$ for all ω_A and ω_B. Hence, Condition (3) is also satisfied.

We provide an illustrative example, and defer the proof to the appendix.

Example 4. There are two inputs, $\Omega_A{=}\{0,1\}$ and $\Omega_B{=}\{0,2\}$, and $v(\omega){=}\omega_A{+}\omega_B$. There are two signals per input, a low signal L_c and a high signal H_c. Conditional on receiving a signal for input c, a trader receives the high signal H_c with probability $\varepsilon \in (0, \frac{1}{2})$ when $\omega_c = 0$, and with probability $1 - \varepsilon$ when $\omega_c \neq 0$. The MLRP is therefore satisfied in each dimension and signals become perfectly informative as $\varepsilon \to 0$. However, $\frac{P(H_A|1,0)}{P(H_B|1,0)}{=}\frac{1-\varepsilon}{\varepsilon}$, which is strictly greater than 1 and diverges to ∞ as $\varepsilon \to 0$; $\frac{P(H_A|0,2)}{P(H_B|0,2)}{=}\frac{\varepsilon}{1-\varepsilon}$, which is strictly less than 1 and converges to 0 as $\varepsilon \to 0$; and $\frac{P(H_A|1,2)}{P(H_B|1,2)}{=}\frac{1-\varepsilon}{1-\varepsilon} = 1$. Since $v(1,0) < v(0,2) < v(1,2)$, the likelihood ratios for the high signals are not monotone in values and so the MLRP is not satisfied (see Figure 9).

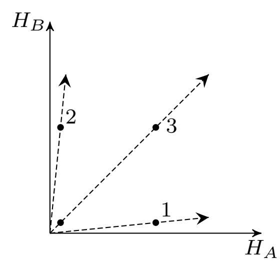


Figure 9: Failure of the MLRP with multiple inputs.

The likelihood ratio between high signals is the ray from the origin passing through the state. For states $(0,2)$, $(1,2)$ and $(1,0)$ likelihood-ratios are not monotone in values.

Even when the MLRP is not satisfied overall, one might conjecture that prices can aggregate information when signals satisfy the MLRP for each input. However, Section 2 provides a counterexample. When there are more signals than states, we can appeal to Proposition 1 to establish generic existence. However, $K \geq M$ is demanding in multi-input environments because information is highly fragmented: with C inputs, there are $\prod_{c=1}^{C} M_c$ states but only $\sum_{c=1}^{C} K_c$ signals. We therefore provide an alternative condition for generic existence, where we only measure the betweenness property in relation to multi-input environments. In that case, a much weaker condition is sufficient for generic information aggregation.

Proposition 5. *In multi-input environments, the betweenness property is generic if and only if $K_c \geq M_c$ for every input c.*

Proposition 5 establishes when information aggregation is a generic equilibrium property in multi-input environments. While the information conveyed by signals is limited, there is additional structure on the states (because of the independence across inputs) and the value function (because of the separability condition). As a result, less information is needed to reveal the value. Multi-input environments therefore provide a stark illustration of our sufficiency result: while the MLRP is never satisfied, there are generic conditions under which equilibrium prices can aggregate information.

8 Conclusion

In this paper, we address a fundamental question of market exchange: when do prices aggregate information? By studying a common value auction with an infinite population of traders, our approach to this question combines insights from both strategic auction and competitive equilibrium models.

Our main result identifies a simple condition on information primitives that is both necessary and sufficient for equilibrium prices to aggregate information. Information aggregation does not require a strong order property on signals, but instead requires an order property on distributions over signals, which we call the betweenness property. While no individual trader observes the conditional distribution, the betweenness property is sufficient for competitive market forces to guide aggregate behavior so that prices are perfectly revealing. On the other hand, when the betweenness property is not satisfied, information aggregation necessarily fails. This highlights the limitations of the market, especially in environments with many states and relatively few signals. In such environments, even if collectively the population is perfectly informed, the market cannot coordinate behavior so that prices reveal the collective information.

A Appendix

The appendix is organized as follows. Section A.1 provides preliminary results on nested hyperplanes. Section A.2 shows that any quasi-linear function can be obtained as the extended price-function of some strategy. Section A.3 establishes the no-arbitrage property. Section A.4 proves the main result (Theorem 1). Section A.5 proves the genericity and multi-input results (Propositions 1–5).

A.1 Preliminaries

For a vector $\alpha \in \mathbb{R}^K$, let $\alpha(k)$ denote the k-th component of α; $\mathbf{0} \equiv (0, ..., 0)$ is the origin; $\mathbf{e} \equiv (1, ..., 1)$ is the vector of 1's; and $\mathbf{e}_k$ is the unit vector with $\mathbf{e}_k(j) = \mathbb{1}[j = k]$, where $\mathbb{1}[.]$ is the indicator function.

For a set $A \subset \mathbb{R}^K$, $co(A)$ denotes the convex hull of A, $cl(A)$ is the closure of A, $\mathring{A}$ is the relative interior of A, and $A^\Delta \equiv A \cap \Delta$ is the intersection of A with the unit-simplex $\Delta = \{x \in \mathbb{R}^K : \mathbf{0} \leq x,\ \mathbf{e} \cdot x = 1\}$.

A.1.1 Intersection of hyperplanes and the unit-simplex

For a vector $\alpha \in \mathbb{R}^K / \{\mathbf{0}\}$ and scalar $c \in \mathbb{R}$, $H(\alpha, c) \equiv \{x : \alpha \cdot x = c\}$ is the hyperplane in $\mathbb{R}^K$ with norm α and constant c; $H_+(\alpha, c)$ is the upper and $H_-(\alpha, c)$ the lower half-space. When $c = 0$, we omit c from the notation (e.g., $H(\alpha) \equiv H(\alpha, 0)$).

Let $\mathcal{H}^\Delta$ be the set of all hyperplanes in $\mathbb{R}^K$ that intersect the unit simplex, i.e., $H(\alpha, c) \in \mathcal{H}^\Delta \Leftrightarrow H^\Delta(\alpha, c) \neq \emptyset$. We define a preorder $\succsim_\Delta$ on $\mathcal{H}^\Delta$ by

$$H(\alpha, c) \succsim_\Delta H(\alpha', c') \quad \Leftrightarrow \quad H_+^\Delta(\alpha, c) \supset H_+^\Delta(\alpha', c'),$$

with symmetric part denoted $\sim_\Delta$. The following lemmas establish properties of $\succsim_\Delta$.

Lemma 1. *Let $\phi \neq 0$: (i) $H(\alpha, c) \sim_\Delta H(\phi\alpha, \phi c)$, and (ii) $H(\alpha, c) \sim_\Delta H(\alpha + \phi\mathbf{e}, c + \phi)$.*

Proof. (i) is trivial and (ii) follows because, for $x \in \Delta$, $(\alpha + \phi\mathbf{e}) \cdot x = \alpha \cdot x + \phi\mathbf{e} \cdot x$ $= \alpha \cdot x + \phi$, and $\alpha \cdot x + \phi \geq c + \phi$ iff $\alpha \cdot x \geq c$. □

Lemma 2. *Let $H_+^\Delta(\alpha') \cap H_-^\Delta(\alpha) \neq \Delta$: $H(\alpha') \succsim_\Delta H(\alpha)$ iff $\lambda\alpha' \geq \alpha$ for some $\lambda > 0$.*

Proof. Suppose $H_+^\Delta(\alpha') \neq \Delta$. We first show that $H(\alpha') \succsim_\Delta H(\alpha)$ implies existence of $\lambda > 0$ such that $\lambda\alpha' \geq \alpha$. We argue the contrapositive: suppose there is no $\lambda > 0$ such that $\lambda\alpha' \geq \alpha$. Then we want to show that there is some $x \in \Delta$ with $x \cdot \alpha \geq 0 > x \cdot \alpha'$. By assumption, $\alpha' \notin Z \equiv \{\tilde{z} \in \mathbb{R}^K : \lambda\tilde{z} \geq \alpha,\ \text{for some } \lambda > 0\}$. Since Z is closed and convex, by the Separating Hyperplane Theorem, there is some $z \in \mathbb{R}^K / \{\mathbf{0}\}$ such that $z \cdot \alpha' < 0 \leq z \cdot \tilde{z}$ for all $\tilde{z} \in Z$. Furthermore, $z \geq 0$. If not, then $z \cdot \mathbf{e}_k < 0$ for some k, and we can argue to the following contradiction: if $\tilde{z} \in Z$, then $z' = \tilde{z} + t\mathbf{e}_k \in Z$ for $t > 0$; but $z \cdot (\tilde{z} + t\mathbf{e}_k)$ can be made arbitrarily small by increasing t, thereby contradicting that $z \cdot z' \geq 0$. Since $z > 0$, we can normalize z so that $z \cdot \mathbf{e} = 1$, i.e.,

$z \in \Delta$. As $\alpha \in Z$, $z \cdot \alpha \geq 0$ (because $\tilde{z} \cdot \alpha \geq 0$ for all $\tilde{z} \in Z$), and so $z \in H_+(\alpha)$. But $z \cdot \alpha' < 0$, and so $z \notin H_+(\alpha')$. Hence, $H_+^\Delta(\alpha)$ is not a subset of $H_+^\Delta(\alpha')$.

For the converse, suppose $\lambda\alpha' \geq \alpha$ for some $\lambda > 0$. It suffices to show that $\lambda\alpha' \cdot z \geq 0$ whenever $z \in H_+^\Delta(\alpha)$ (since this implies that $\alpha' \cdot z \geq 0$). To see this, note that $\lambda\alpha' \cdot z = \alpha \cdot z + (\lambda\alpha' - \alpha) \cdot z$. The first term is non-negative because $z \in H_+(\alpha)$. The second term is non-negative because $(\lambda\alpha' - \alpha) \geq 0$ by assumption, and $z \geq 0$. As a result, $z \in H_+^\Delta(\alpha)$ implies $z \in H_+(\alpha')$.

Now suppose $H_-^\Delta(\alpha) \neq \Delta$ and $H(\alpha') \succsim_\Delta H(\alpha)$. Then $H(-\alpha) \succsim_\Delta H(\alpha')$. Moreover, $H_+^\Delta(-\alpha) \neq \Delta$. By the previous argument, there is some $\frac{1}{\lambda} > 0$ such that $\frac{1}{\lambda}(-\alpha) \geq -\alpha'$, hence $\lambda\alpha' \geq \alpha$. On the other hand, if $\lambda\alpha' \geq \alpha$ for some $\lambda > 0$, then $\frac{1}{\lambda}(-\alpha) \geq -\alpha'$ for $\frac{1}{\lambda} > 0$, and so by the previous argument, $H(-\alpha) \succsim_\Delta H(-\alpha')$, which implies $H(\alpha') \succsim_\Delta H(\alpha)$. □

Lemma 3. *Let $\bar{\alpha} \geq \underline{\alpha}$ and $H_+^\Delta(\bar{\alpha}) \cap H_-^\Delta(\underline{\alpha}) \neq \Delta$: $H(\bar{\alpha}) \succsim_\Delta H(\alpha') \succsim_\Delta H(\underline{\alpha})$ iff $H(\alpha') \sim_\Delta H(\alpha)$ for some $\bar{\alpha} \geq \alpha \geq \underline{\alpha}$.*

Proof. We focus on the case where $H_+^\Delta(\bar{\alpha}) \neq \Delta$. The other case is symmetric, replacing $(\bar{\alpha}, \underline{\alpha})$ with $(-\bar{\alpha}, -\underline{\alpha})$ as in the proof of Lemma 2. When $H_+^\Delta(\bar{\alpha}) \neq \Delta$, then $\bar{\alpha}(k) < 0$ for some $k \in \{1, ..., K\}$ because $\alpha \cdot x < 0$ for some $x \in \Delta$. As a result, if $\bar{\alpha} \geq \hat{\alpha}$, then $\hat{\alpha}(k) < 0$. In the remaining proof, all vectors are dominated by $\bar{\alpha}$, and are therefore bounded away from 0 in at least one component.

If $\bar{\alpha} \geq \alpha \geq \underline{\alpha}$, then it follows from Lemma 2 that $H(\bar{\alpha}) \succsim_\Delta H(\alpha) \succsim_\Delta H(\underline{\alpha})$. For the converse, first consider any $\tilde{\alpha}$ such that $H_+^\Delta(\tilde{\alpha}) \neq \Delta$, $\bar{\alpha} \geq \tilde{\alpha} \geq \underline{\alpha}$ and $H(\tilde{\alpha}) \succsim_\Delta H(\alpha') \succsim_\Delta H(\underline{\alpha})$. At least one such $\tilde{\alpha}$ exists since $\bar{\alpha}$ satisfies these conditions. For any k, let $A^k(\tilde{\alpha}) = \{\underline{\alpha} \leq \alpha \leq \tilde{\alpha} : \alpha(k') = \tilde{\alpha}(k') \text{ for } k' \neq k, H(\alpha) \succsim_\Delta H(\alpha')\}$. The set $A^k(\tilde{\alpha})$ has the following properties: (i) it is non-empty because it contains $\tilde{\alpha}$, (ii) it is convex by Lemma 2, (iii) it is closed because weak inequalities are preserved in the limit,[12] (iv) it is totally ordered by the vector order $\geq$ on $\mathbb{R}^K$ because its elements differ only in the k-th component. As a result, $A^k(\tilde{\alpha})$ has a unique least element $\alpha^k(\tilde{\alpha}) \in A^k(\tilde{\alpha})$ such that $\alpha \geq \alpha^k(\tilde{\alpha})$ for all $\alpha \in A^k(\tilde{\alpha})$.

Finally, we construct a sequence $\{\alpha_n\}_{n=0}^\infty$ in $\mathbb{R}^K/\{\mathbf{0}\}$ with a non-zero limit that has the desired properties. In particular, let $\alpha_0 = \bar{\alpha}$ and, for $n \geq 1$, let $\alpha_n = \alpha^k(\alpha_{n-1})$

[12] Consider a sequence $\alpha_n \in A^k(\tilde{\alpha})$ with limit α^*. Then $\alpha^*(k') = \tilde{\alpha}(k')$ for $k' \neq k$ since $\alpha_n(k') = \tilde{\alpha}(k')$ for all n, and $\tilde{\alpha}(k) \geq \alpha^*(k) \geq \underline{\alpha}_k$ since $\tilde{\alpha}(k) \geq \alpha_n(k) \geq \underline{\alpha}_k$ for all n. Moreover, for any point $x \in H_+^\Delta(\alpha')$, $\alpha^* \cdot x \geq 0$ because $\alpha_n \cdot x \geq 0$ for all n. Hence, $\alpha^* \in A^k(\tilde{\alpha})$.

iff $k \equiv n$ (modular K). By construction, the sequence $\{\alpha_n\}$ is monotone decreasing in the vector order $\geq$ and bounded below by $\underline{\alpha}$, and so the Monotone Convergence Theorem implies $\{\alpha_n\}$ has a limit α^*. Since weak inequalities are preserved in the limit, $\bar{\alpha} \geq \alpha^* \geq \underline{\alpha}$ and $H(\alpha^*) \succsim_\Delta H(\alpha')$ (cf. footnote 12). Moreover, $H(\alpha') \succsim_\Delta H(\alpha^*)$, otherwise Lemma 2 implies that $\alpha^*(k) < \alpha'(k)$ for some k, which contradicts that α^* is the limit of $\{\alpha_n\}$. Defining $\alpha \equiv \alpha^*$ completes the proof. □

A.2 Extended price-function

For strategy-profile $\sigma : \mathcal{I} \times S \to \mathcal{B}$, we define an *extended price-function* $\tilde{p}_\sigma : \Delta \to B$ by $\tilde{p}_\sigma(x) = \mathcal{Q}^\sigma_x(1-\kappa)$. In Section 5.1, we show that the extended price-function for any strategy-profile σ is quasi-linear. To establish a converse, the following lemma first provides a geometric characterization of quasi-linear functions.

Lemma 4. *Suppose $V : \Delta \to B$ satisfies $|V(\Delta)| \geq 3$, then the following statements are equivalent:*

(1) V is quasi-linear.

(2) There is a lower semicontinuous mapping from Δ to $\mathbb{R}^K/\{\mathbf{0}\}$, $x \mapsto \alpha_x$, such that (i) $y \cdot \alpha_x \geq 0$ if and only $V(y) \geq V(x)$ and (ii) $\alpha_x \geq \alpha_y$ if and only if $V(x) \geq V(y)$.

Proof. The direction (2) implies (1) is trivial and so we omit the proof. For the converse, suppose that $V : \Delta \to B$ is quasi-linear.

For $x \in \Delta$, let $p_x \equiv V(x)$, $[x]_\geq \equiv \{y : p_y \geq p_x\}$, and define $[x]_=$, $[x]_\leq$, $[x]_>$ and $[x]_<$ analogously. For $s \in S$, let $p_s \equiv V(\delta_s)$. Let $\underline{s} \in \underline{S} = \{s \in S : p_x \geq p_s \ \forall x\}$ and $\bar{s} \in \bar{S} = \{s \in S : p_s \geq p_x \ \forall x\}$; since $\Delta = co\{\delta_s : s \in S\}$, the sets $\underline{S}$ and $\bar{S}$ are non-empty by the betweenness property of V. Now let $x \in [\delta_{\bar{s}}]_<$. Since $p_{\bar{s}} > p_x \geq p_{\underline{S}}$, the sets $[x]_\leq$ and $[x]_>$ are non-empty; both sets are convex by the betweenness property; $[x]_\leq$ is closed and $[x]_>$ relatively open because V is lower semi-continuous. By the Separating Hyperplane Theorem, there are $\alpha''_x \in \mathbb{R}^K/\{\mathbf{0}\}$ and $c_x \in \mathbb{R}$ such that $\alpha''_x \cdot y \leq c_x < \alpha''_x \cdot y'$ for all $y \in [x]_\leq$ and $y' \in [x]_>$. Let $\alpha'_x = \alpha''_x - \mathbf{e}c_x$; this is a non-zero vector because otherwise $\alpha''_x \cdot y = c_x$ for all $y \in \Delta$. Then, $[x]_\leq = H^\Delta_-(\alpha'_x)$ and $[x]_> = \mathring{H}^\Delta_+(\alpha'_x)$. For $x \in [\delta_{\bar{s}}]_=$, let $\alpha'_x = -\sum_{\{k : s_k \notin \bar{S}\}} \mathbf{e}_k$; then $\alpha_x \cdot \delta_s = 0 > \alpha_x \cdot y$ for all $s \in \bar{S}$ and $y \in \Delta/co\{\delta_s : s \in \bar{S}\}$.

Observe that $p_x \geq p_y$ iff $H(\alpha'_y) \succsim_\Delta H(\alpha'_x)$. First, suppose $p_x \geq p_y$. If $\alpha'_x \cdot z > 0 \geq \alpha'_y \cdot z$ for some $z \in \Delta$; then $p_z > p_x$ and $p_y \geq p_z$, contradicting

$p_x \geq p_y$. Hence, for all $z \in \Delta$, $\alpha'_x \cdot z > 0$ implies $\alpha'_y \cdot z > 0$; or equivalently $\{z \in \Delta : \alpha'_x \cdot z > 0\} \subset \{z \in \Delta : \alpha'_y \cdot z > 0\}$. Hence, $H^\Delta_+(\alpha'_x) = cl(\{z \in \Delta : \alpha'_x \cdot z > 0\}) \subset cl(\{z \in \Delta : \alpha'_y \cdot z > 0\}) = H^\Delta_+(\alpha'_y)$. On the other hand, if $H(\alpha'_y) \succsim_\Delta H(\alpha'_x)$, then either $H(\alpha'_y) \sim_\Delta H(\alpha'_x)$, in which case $p_x = p_y$, or there is $z \in \mathring{H}^\Delta_+(\alpha'_y) \cap H^\Delta_-(\alpha'_x)$, and so $p_x \geq p_z > p_y$.

Since $|V(\Delta)| \geq 3$, there is $\hat{x} \in \Delta$ such that $p_{\bar{s}} > p_{\hat{x}} > p_{\underline{S}}$, and so $H^\Delta_+(\alpha'_{\hat{x}}) \neq \Delta \neq H^\Delta_-(\alpha'_{\hat{x}})$. Let $\hat{\alpha} = \alpha'_{\hat{x}}$. Then, $H(\alpha'_{\delta_{\underline{S}}}) \succsim_\Delta H(\hat{\alpha}) \succsim_\Delta H(\alpha'_{\delta_{\bar{s}}})$ and, by Lemma 2, there are $\lambda, \lambda' > 0$ such that $\lambda\alpha'_{\delta_{\underline{S}}} \geq \hat{\alpha} \geq \lambda'\alpha'_{\delta_{\bar{s}}}$. Let $\bar{\alpha} = \lambda\alpha'_{\delta_{\underline{S}}}$ and $\underline{\alpha} = \lambda'\alpha'_{\delta_{\bar{s}}}$.

Since Δ is a compact subset of the separable metric space $\mathbb{R}^K$, it has a countable dense subset $\{x_n\} \equiv \{x_n \in \Delta : n \in \mathbb{N}\}$. Without loss of generality, let $x_1 = \delta_{\bar{s}}$, $x_2 = \delta_{\underline{S}}$ and $x_3 = \hat{x}$. We use Lemma 3 to normalize the norms $\{\alpha_{x_n}\}$, and then extend to Δ by taking limits. We proceed by induction. Step 1: let $\alpha_{x_1} = \underline{\alpha}$, $\alpha_{x_2} = \bar{\alpha}$, and $\alpha_{x_3} = \hat{\alpha}$. Step $n > 1$: if $p_{x_n} = p_{x_m}$ for some $m < n$, let $\alpha_{x_n} = \alpha_{x_m}$. Otherwise, there are $m, m' < n$ such that (i) $p_{x_m} > p_{x_n} > p_{x_{m'}}$, (ii) $\ell < n$ and $p_{x_\ell} > p_{x_n}$ implies $p_{x_\ell} \geq p_{x_m}$, (iii) $\ell' < n$ and $p_{x_n} > p_{x_{\ell'}}$ implies $p_{x_{m'}} \geq p_{x_{\ell'}}$, and (iv) either $H^\Delta_+(\alpha_{x_m}) \neq \Delta$ or $H^\Delta_+(\alpha_{x'_m}) \neq \Delta$. Moreover, $H(\alpha_{x_{m'}}) \succsim_\Delta H(\alpha'_{x_n}) \succsim_\Delta H(\alpha_{x_m})$, and by Lemma 3, there is α^*_n such that $\alpha_{x_{m'}} \geq \alpha^*_n \geq \alpha_{x_m}$ and $H(\alpha^*_n) \sim_\Delta H(\alpha'_{x_n})$. Let $\alpha_{x_n} = \alpha^*_n$. This process of induction assigns α_x to every $x \in \{x_n\}$.

We now extend the construction to Δ by taking limits. Suppose $x \notin \{x_n\}$. We first show that $H^\Delta_+(\alpha'_x) = cl(\bigcup_{\{n:p_{x_n} \geq p_x\}} H^\Delta_+(\alpha_{x_n}))$. If $p_x \geq p_{x_n}$ for some n, then $H(\alpha'_x) \sim H(\alpha_{x_n})$, and it is trivial. More generally, since $p_x \geq p_{x_n}$, $H^\Delta_+(\alpha'_x) \supset H^\Delta_+(\alpha_{x_n})$, and so $H^\Delta_+(\alpha'_x) \supset cl(\bigcup_{\{n:p_{x_n} \geq p_x\}} H^\Delta_+(\alpha_{x_n}))$ because $H^\Delta_+(\alpha'_x)$ is closed. We now show that $\mathring{H}^\Delta_+(\alpha'_x) \subset cl(\bigcup_{\{n:p_{x_n} \geq p_x\}} H^\Delta_+(\alpha_{x_n}))$. For contradiction, suppose not: there is $z \in \Delta$ such that $\alpha'_x \cdot z > 0$ and $z \notin H^\Delta_+(\alpha_{x_n})$ for any $p_{x_n} \leq p_x$. Now consider $Z = \{z' \in \Delta : \alpha'_x \cdot z' > 0 > \alpha'_z \cdot z'\}$. This set is open, and it is non-empty because otherwise $H^\Delta_+(\alpha'_x) = cl(\{z'' \in \Delta : \alpha'_x \cdot z'' > 0\}) = cl(\{z''' \in \Delta : \alpha'_y \cdot z''' < 0\}) = H^\Delta_-(\alpha'_z)$ and so $H(\alpha'_x) \sim_\Delta H(\alpha'_z)$, which would imply $p_z = p_x$. Since $\{x_n\}$ is dense, there is some $x_m \in Z \cap \{x_n\}$. Since $\alpha'_z \cdot x_m < 0$, $p_z \geq p_{x_m}$, and so $z \in H^\Delta_+(\alpha_{x_m})$. But since $\alpha'_x \cdot x_m > 0$, $p_{x_m} > p_x$, yielding a contradiction. As a result, it must be that $\mathring{H}^\Delta_+(\alpha'_x) \subset cl(\bigcup_{\{n:p_{x_n} \geq p_x\}} H^\Delta_+(\alpha_{x_n}))$ and so $cl(\mathring{H}^\Delta_+(\alpha'_x)) \subset cl(\bigcup_{\{n:p_{x_n} \geq p_x\}} H^\Delta_+(\alpha_{x_n}))$.

The collection $\{H^\Delta_+(\alpha_{x_n}) : p_{x_n} \geq p_x\}$ is nested: for n, m, $H^\Delta_+(\alpha_{x_n}) \subset H^\Delta_+(\alpha_{x_m})$ or $H^\Delta_+(\alpha_{x_m}) \subset H^\Delta_+(\alpha_{x_n})$. Hence, there exists a sequence $\{x_{n_r}\}_{r=1}^\infty$ in $\{x_n : p_{x_n} \geq p_x\}$ such that $H^\Delta_+(\alpha_{x_{n_r}}) \subset H^\Delta_+(\alpha_{x_{n_{r+1}}})$, and $\bigcup_{\{n:p_{x_n} \geq p_x\}} H^\Delta_+(\alpha_{x_n}) = \lim_{r\to\infty} H^\Delta_+(\alpha_{x_{n_r}})$. By construction, the sequence $\{\alpha_{x_{n_r}}\}_{r=1}^\infty$ is monotone increasing in the vector order

$\geq$ on $\mathbb{R}^K$, and bounded above by $\bar{\alpha}$, hence it has a limit α_x. By the argument in the previous paragraph $H(\alpha'_x) \sim_\Delta H(\alpha_x)$. Finally, note that if $p_x \geq p_y$, then the sequence $\{x_{n_r}\}$ for obtaining α_x contains the sequence for obtaining α_y, and so $\alpha_x \leq \alpha_y$. We therefore have, for all $x, y \in \Delta$, $p_x \geq p_y$ iff $\alpha_x \leq \alpha_y$. Since, by construction, $\alpha_x \cdot x = 0$ for all x, it follows immediately that $y \cdot \alpha_x \geq 0$ iff $p_y \geq p_x$. □

The following proposition shows that any quasi-linear function $V : \Delta \to B$ can be viewed as the extended price-function of a strategy-profile in the auction mechanism.

Proposition 6. *If the function $V : \Delta \to B$ is quasi-linear, then there is a strategy-profile σ with $\tilde{p}_\sigma \equiv V$.*

Proof. Following the notation in the proof of Lemma 4, let $p_x = V(x)$ for all $x \in \Delta$. If $p_x = c$ for all $x \in \Delta$ the proof is trivial: let $\sigma(i, s) = \delta_c$ for all types, then $\tilde{p}_\sigma(x) = c = V(x)$. The case where V is binary is covered in Section 5.1. We therefore focus on $|V(\Delta)| \geq 3$. In that case, Lemma 4 shows that there is a mapping $x \mapsto \alpha_x$ such that (i) $y \cdot \alpha_x \geq 0$ iff $p_y \geq p_x$, and (ii) $p_x \geq p_y$ iff $\alpha_x \leq \alpha_y$.

We need a further adjustment to the non-zero vectors representing V. First, let $\underline{c} = \min_k \alpha_{\bar{s}}(k)$, which is strictly negative (see proof of Lemma 3), $\bar{c} = \max_k \alpha_{\underline{S}}(k)$, which is strictly positive since $\alpha_{\underline{S}} \cdot \delta_{\bar{s}} > 0$, and finally let $c = \max\{\kappa/|\underline{c}|, (1-\kappa)/|\bar{c}|\}$. For each $x \in [\delta_{\bar{s}}]_\prec$, let $\tilde{\alpha}_x = c\alpha_x$. Since $c > 0$, Lemma 1 implies $H(\tilde{\alpha}_x) \sim_\Delta H(\alpha_x)$. Moreover, $\tilde{\alpha}_x \geq \tilde{\alpha}_y$ if and only if $\alpha_x \geq \alpha_y$. Since $\underline{\alpha} \geq \alpha_x \geq \bar{\alpha}$, we also have that $-\kappa \leq \tilde{\alpha}_x(k) \leq 1-\kappa$ for $k = 1, ..., K$. Second, let $\alpha^*_x = -\tilde{\alpha}_x + (1-\kappa)\mathbf{e}$. By Lemma 1, $H(\alpha_x) \sim_\Delta H(\alpha^*_x, 1-\kappa)$, and $\alpha^*_x \geq \alpha^*_y$ iff $\alpha_x \leq \alpha_y$ iff $p_x \geq p_y$. Moreover, $\mathbf{e} \geq \alpha^*_x \geq \mathbf{0}$ for all x. Finally, let $\alpha^*_{\delta_{\bar{s}}} = \mathbf{e}$, which preserves the full order.

For the prices, define the mapping $g : B \to B$ by $g(b) = \max_{x \in \Delta}\{p_x : p_x \leq b\}$ if $b \geq p_{\underline{S}}$ and $g(b) = p_{\underline{S}}$ otherwise; the maximum exists when $b \geq p_{\underline{S}}$ because V is lower semicontinuous. Then g is right-continuous: suppose $g(b) < g(b')$ and let $p_y = g(b)$ and $p_x = g(b')$. Since $p_x > p_y$, it is without loss of generality to let $x \in H(\alpha_x)$ and $y \in H(\alpha_y)/H(\alpha_x)$ (since $H^\Delta(\alpha_x) \neq H^\Delta(\alpha_y)$, and $p_x = g(b')$, $p_y = g(b')$ for all $x \in H(\alpha_x)$ and $y \in H(\alpha_y)$). Then, $\alpha_x \cdot x = 0 > \alpha_x \cdot y$ and $\alpha_y \cdot x > 0 = \alpha_y \cdot y$. Let $z = \frac{1}{2}x + \frac{1}{2}y$, then $\alpha_x \cdot z < 0 < \alpha_y \cdot z$. Hence, $p_z \in (p_y, p_x)$, and so $g(b'') \in (p_y, p_z)$ for some $b'' \in (b, b')$. It follows that for any sequence $b_n \downarrow b$, $g(b_n) \downarrow g(b)$.

We can now construct the strategy-profile. For $k = 1, ..., K$, define the function $F_{s_k} : B \to \mathbb{R}$ by $F_{s_k}(b) = \alpha^*_x(k)$ if and only if $g(b) = p_x$. This function is well-defined since $p_y = p_x$ implies $\alpha^*_y = \alpha^*_x$, and takes values in $[0,1]$, attaining 1 at $\bar{b}$. The function is monotone increasing because $b \geq b'$ implies $g(b) \geq g(b')$, and $p_x \geq p_y$ implies $\alpha^*_x(k) \geq \alpha^*_y(k)$. Finally, we argue that the function is right-continuous. Consider a sequence $\{b_r\}_{r=1}^{\infty}$ with $b_r \downarrow b$; we need to show that $\lim_{r\to\infty} F_{s_k}(b_r) = F_{s_k}(b)$. Let $p_x = g(b)$ and $\{x_r\}_{r=1}^{\infty}$ be a sequence in Δ such that $p_{x_r} = g(b_r)$ for $r = 1, ..., \infty$. Since g is right-continuous, $\lim_{r\to\infty} g(b_r) = g(b)$. Since $b_r \geq b$, $g(b_r) \geq g(b)$, and so $p_{x_r} \geq p_x$ for all r; hence $\alpha^*_{x_r} \geq \alpha^*_x$. If $p_{x_r} = p_x$ for some r, then $g(b_r) = g(b)$ and so $F_{s_k}(b_r) = F_{s_k}(b)$. Otherwise, $p_{x_r} > p_x$ for all r, and we can argue as in the extension of $\{x_n\}$ to Δ in Lemma 4: there is a sequence $\{x_{n_r}\}$ in $\{x_n\}$ such that $p_{x_r} > p_{x_{n_r}} > p_x$ for all r. By the construction, $\alpha_x(k) = \lim_{r=1}^{\infty} \alpha_{x_{n_r}}(k)$, and so $F_{s_k}(b_r) \downarrow F_{s_k}(b)$. As a result, F_{s_k} is a cumulative distribution function for some Borel-measure $\beta(s_k) \in \mathcal{B}$.

Consider the symmetric strategy-profile $\sigma : \mathcal{I} \times S \to \mathcal{B}$, where $\sigma(i, s_k) = \beta(s_k)$ for all $i \in \mathcal{I}$ and $k = 1, ..., K$. Since the strategy-profile is symmetric, $F^{\sigma}_{s_k}(b) = \alpha^*_x(k)$ if and only if $g(b) = p_x$. It remains to verify that $\tilde{p}_\sigma(x) = p_x$ for all $x \in \Delta$. If $g(b) \geq p_x$, then $g(b) = p_y$ for some y with $p_y \geq p_x$. Hence, $F^{\sigma}(b) = \alpha^*_y$ and $\alpha^*_y \cdot x \geq 1 - \kappa$. If $g(b') < p_x$, then $g(b') = p_z$ for some z with $p_z < p_x$. Hence, $F^{\sigma}(b) = \alpha^*_z$ and $\alpha^*_z \cdot x < 1 - \kappa$. As a result, p_x is the $(1-\kappa)$-quantile of F^{σ}_x. □

A.3 The no-arbitrage property

Proposition 7. *The strategy-profile σ is an equilibrium that aggregates information if and only if $p_\sigma(\omega) = v(\omega)$ for every state ω.*

Proof. "If" is trivial and so we focus on "only if." We proceed by contradiction: suppose σ is an equilibrium that aggregates information but $p_\sigma \neq v$. We consider the case where $p_\sigma(\omega) < v(\omega)$ for some ω; the argument when the asset is overpriced is symmetric. Let $p \equiv p_\sigma(\omega)$, $v \equiv v(\omega)$, and $\varepsilon = \frac{1}{2}\min\{|p - p_\sigma(\omega')| : p_\sigma(\omega') \neq p\} > 0$. Let $\varepsilon' = \min\{\varepsilon, p, \bar{b} - p\}$. Note $\bar{b} - p > 0$ because $p < v(\omega) \leq \bar{b}$.

First, suppose $p > 0$, so $\varepsilon' > 0$. For type (i, s), consider deviation $\sigma'(i, s)$ where

$$F^{\sigma'(i,s)}(b) = \begin{cases} F^{\sigma(i,s)}(p - \varepsilon') & \text{if } b \in [p - \varepsilon', p + \varepsilon') \\ F^{\sigma(i,s)}(b) & \text{otherwise} \end{cases}.$$

By construction, deviating from $\sigma(i, s)$ to $\sigma'(i, s)$ does not affect the probability that trader i wins the auction in any state where the price differs from p. Moreover, since

σ aggregates information, the value is equal to v in any state where the price equals p. We now consider two cases.

(i) If $F^\sigma_\omega(p) > \vec{F}^\sigma_\omega(p)$, then let $\phi = \frac{F^\sigma_\omega(p)-(1-\kappa)}{F^\sigma_\omega(p)-\vec{F}^\sigma_\omega(p)}$, which is the probability that trader i wins the auction with a bid of p in state ω. The expected payoff difference between $\sigma'(i,s)$ and $\sigma(i,s)$ is greater than equal to[13]

$$P_s(\omega)(v-p)\left(-\phi(F^{\sigma(i,s)}(p) - \vec{F}^{\sigma(i,s)}(p)) + (F^{\sigma(i,s)}(p) - F^{\sigma(i,s)}(p-\varepsilon'))\right) \geq 0$$

Since σ is an equilibrium, the payoff difference must equal 0 $\lambda-a.s.$ Therefore, integrating over $\mathcal{I}$, $F^\sigma_s(p) - F^\sigma_s(p-\varepsilon') = \phi(F^\sigma_s(p) - \vec{F}^\sigma_s(p))$. Since this holds for all s with $P_\omega(s) > 0$, $F^\sigma_\omega(p) - F^\sigma_\omega(p-\varepsilon') = \phi(F^\sigma_\omega(p) - \vec{F}^\sigma_\omega(p)) = F^\sigma_\omega(p) - (1-\kappa)$, which implies $F^\sigma_\omega(p-\varepsilon) = 1-\kappa$, contradicting $p = \mathcal{Q}^\sigma_\omega(1-\kappa)$.

(ii) If $F^\sigma_\omega(p) = \vec{F}^\sigma_\omega(p)$, the expected payoff difference between $\sigma'(i,s)$ and $\sigma(i,s)$ is greater than equal to $P_s(\omega)(v-p)\left(F^{\sigma(i,s)}(p) - F^{\sigma(i,s)}(p-\varepsilon')\right) \geq 0$. Again, equilibrium implies equality $\lambda-a.s.$. Integrating over $\mathcal{I}$, $F^\sigma_s(p) = F^\sigma_s(p-\varepsilon)$. Since this holds for all s, $F^\sigma_\omega(p) = F^\sigma_\omega(p-\varepsilon')$, contradicting $p = \mathcal{Q}^\sigma_\omega(1-\kappa)$.

Now suppose $p = 0$, so $F^\sigma_\omega(0) \geq 1-\kappa$. For type (i,s), consider deviation $\sigma''(i,s)$:

$$F^{\sigma''(i,s)}(b) = \begin{cases} 0 & \text{if } b < p + \varepsilon \\ F^{\sigma(i,s)}(b) & \text{otherwise} \end{cases}.$$

Again, deviating from $\sigma(i,s)$ to $\sigma'(i,s)$ does not affect the probability that trader i wins the auction in any state where the price differs from p. Let $\psi = \frac{F^\sigma_\omega(0)-(1-\kappa)}{F^\sigma_\omega(0)}$, which is the probability that trader i wins in state ω with a bid of p, and note that $\psi \in [0,\kappa]$. The expected payoff difference between $\sigma''(i,s)$ and $\sigma(i,s)$ is greater than equal to $P_s(\omega)(v-p)(1-\psi)F^{\sigma(i,s)}(0)$. In equilibrium, equality must hold $\lambda-a.s.$ Therefore, integrating over $\mathcal{I}$, $F^\sigma_s(0) = 0$. Since this holds for all s with $P_\omega(s) > 0$, it follows that $F^\sigma_\omega(0) = 0$, contradicting $p = \mathcal{Q}^\sigma_\omega(1-\kappa)$. □

A.4 Proof of Theorem 1

Proof. Given Propositions 6 and 7, and the arguments in Section 5.1, it remains to generalize the re-scaling of bids to the case where v is not injective. For this, let

[13] The first two terms are strictly positive by assumption. The term in the last bracket is non-increasing in ϕ and takes value value $\vec{F}^{\sigma(i,s)}(p) - F^{\sigma(i,s)}(p-\varepsilon') \geq 0$ when $\phi = 1$.

$\{\omega_1, ..., \omega_N\} \subset \mathbf{\Omega}$ such that (i) $v(\omega_n) < v(\omega_{n+1})$ for all $n = 1, ..., N$, (ii) for every state ω, $v(\omega) = v(\omega_n)$ for some $n = 1, ..., N$, and (iii) $p_n \equiv p_\sigma(\omega_n) \geq p_\sigma(\omega)$ whenever $v(\omega) = v(\omega_n)$. Now consider the cumulative distribution for signal s defined by:

$$F^{\hat{\sigma}(i,s)}(b) = \begin{cases} 0 & \text{if } b < v(\omega_1) \\ F_s^\sigma(p_n) & \text{if } b \in [v(\omega_n), v(\omega_{n+1})) \text{ and } n = 1, ..., N-1 \\ 1 & \text{if } b \geq v(\omega_N) \end{cases}.$$

The function $F^{\hat{\sigma}(i,s)}$ is right-continuous by construction, and monotone increasing because $\tilde{p}_\sigma$ is monotone in values. If all types follow this strategy, then $F_s^{\hat{\sigma}} = F^{\hat{\sigma}(i,s)}$. To show prices equal values, first suppose $v(\omega) = v(\omega_1)$. If $b < v(\omega)$, then $F^{\hat{\sigma}}(b) = \mathbf{0}$ and so $F_\omega^{\hat{\sigma}}(b) = 0$. If $b \geq v(\omega)$, then $F^{\hat{\sigma}}(b) \geq F^\sigma(p_1) \geq F^\sigma(p_\sigma(\omega))$ because $p_1 \geq p_\sigma(\omega)$. Therefore, $F_\omega^{\hat{\sigma}}(b) \geq F_\omega^\sigma(p_\sigma(\omega)) \geq 1 - \kappa$. Hence, $p_{\hat{\sigma}}(\omega) = v(\omega)$. Now suppose $v(\omega) > v(\omega_1)$. If $b < v(\omega)$, then $F^{\hat{\sigma}}(b) \leq F^\sigma(p_{n-1})$ and so $F_\omega^{\hat{\sigma}}(b) \leq F_\omega^\sigma(p_{n-1}) < 1-\kappa$ because $p_{n-1} < p_\sigma(\omega)$. If $b \geq v(\omega)$, then $F^{\hat{\sigma}}(b) \geq F^\sigma(p_n) \geq F^\sigma(p_\sigma(\omega))$ because $p_\sigma(\omega) \geq p_n$. Therefore, $F_\omega^{\hat{\sigma}}(b) \geq F_\omega^\sigma(p_\sigma(\omega) \geq 1-\kappa$, and so $p_{\hat{\sigma}}(\omega) = v(\omega)$. □

A.5 Genericity analysis

We first describe how we measure environments to quantify the betweenness property.

Fix the set of states $\mathbf{\Omega}$, the set of signals S, and the value function $v : \mathbf{\Omega} \to B$. Let $\mathcal{P}$ denote the set of all the information structures on $\mathbf{\Omega} \times S$ such that P_ω has full support for every ω. Let $\mathcal{P}_B \subset \mathcal{P}$ be the subset of information structures so that the betweenness property is satisfied given v, and $\mathcal{P}_M \subset \mathcal{P}$ those that satisfy the MLRP. The sets $\mathcal{P}$ and $\mathcal{P}_B$ are open in $\mathbb{R}^{(K-1)M}$ and therefore Lebesgue measurable. The boundary of $\mathcal{P}_M$ (the set difference between $\mathcal{P}_M$ and its the closure) is a Lebesgue null-set, and so $\mathcal{P}_M$ is measurable. By the Fubini Theorem, $\mu(\mathcal{P}) = 1$.

For multi-input environments, we fix the number of inputs C, the set of states $\mathbf{\Omega} = \mathbf{\Omega}_1 \times ... \times \mathbf{\Omega}_C$, the set of signals $S = S_1 \cup ... \cup S_C$, the marginal distribution over signals $\gamma = (\gamma_1, ..., \gamma_C)$, and the components ψ and $(\phi_1, ..., \phi_C)$ of the value function $v : \mathbf{\Omega} \to B$. For $c = 1, ..., C$, let Δ_c denote the simplex on S_c. Let $\tilde{\Delta} = \{x \in \Delta : \exists (x_1, ..., x_C) \in \prod_{c=1}^C \Delta_c \text{ s.t. } x(s_c) = \gamma_c x_c(s_c)\ \forall s_c \in S_c, c \in \{1, .., C\}\}$. Then $\tilde{\Delta}$ is closed and convex, and if $(v, \{P_\omega : \omega \in \mathbf{\Omega}\})$ is multi-input, $P_\omega \in \tilde{\Delta}$ for all ω. For $\mathcal{P} \subset \tilde{\Delta}$, let $\mathcal{P}_c = \{x_c \in \Delta_c : x \in \mathcal{P}\}$. We measure $\mathcal{P}$ by $\tilde{\mu}(\mathcal{P}) = \sum_{c=1}^C \gamma_c \mu(\mathcal{P}_c)$. Then

$\tilde{\mu}(\tilde{\Delta}) = \sum_c \gamma_c \mu(\Delta_c) = \sum_c \gamma_c = 1$. Let $\mathcal{P}_B^C$ be the subset of information structures in $\tilde{\Delta}$ for which the betweenness property is satisfied given the value function v.

A.5.1 Proof of Proposition 1

Proof. Given the argument for $K \geq M$ in Section 6, it remains to show that the betweenness property does not have full Lebesgue measure when $K > M$. Assume, without loss of generality, that $v(\omega_1) < ... < v(\omega_M)$. Fix $0 \leq \theta \leq \frac{1}{K}$. For $k = 1, ..., K$, let $A_k = \{z \in \Delta : z(k) \geq 1 - \theta\}$, and $A_{K+1} = \{z \in \Delta : z \geq \theta \mathbf{e}\}$. When $\theta = 1$, $\mu(A_{K+1}) = 0 < \mu(A_k)$; when $\theta = 0$, $\mu(A_k) = 0 < \mu(A_{K+1})$; hence, there is $\bar{\theta} \in (0, 1)$ such that $\mu(A_k) = \mu(A_{K+1})$ for all k. Now let E_k be the event that $P_{\omega_k} \in A_k$ for $k = 1, ..., K + 1$. Then $\mu(\bigcap_{k=1}^{K+1} E_k) = \left(\bar{\theta}\right)^{K+1} > 0$. But on the event E, $P_{\omega_{K+1}} \in \text{co}\{P_{\omega_1}, ..., P_{\omega_K}\}$, which is inconsistent with the betweenness property. □

A.5.2 Proof of Proposition 2

Proof. We adapt the argument from Proposition 1 as follows. Fix the number of signals K and define $A_1, ..., A_{K+1}$ as before. For any $M > K$, let n_M be the largest integer such that $n_M(K + 1) \leq M$ and, without loss of generality, let $Q_1, ..., Q_{K+1}$ be a partition of the states such that $|Q_k| = n_M$ for all $k = 1, ..., K$, and $(\omega, \omega') \in Q_k \times Q_{k+1}$ implies $v(\omega) < v(\omega')$ for $k = 1, ..., K + 1$. For all k, let E_k be the event that $P_\omega \in A_k$ for some $\omega \in Q_k$. By the binomial formula, for $k \leq K$, $\mu(E_k) = \sum_{x=1}^{n_M} \binom{n_M}{x} \bar{\theta}^x (1 - \bar{\theta})^{n_m - x} = 1 - (1 - \bar{\theta})^{n_M}$, and $\mu(E_{K+1}) = \sum_{x=1}^{M-Kn_M} \binom{M-Kn_M}{x} \bar{\theta}^x (1 - \bar{\theta})^{M-Kn_m-x} = 1 - (1 - \bar{\theta})^{M-Kn_M}$. Since $|Q_{K+1}| \geq n_M$, $\mu(E_{K+1}) \geq 1 - (1 - \bar{\theta})^{n_M}$. The betweenness property is not satisfied on the event $E = \bigcap_{k=1}^{K+1} E_k$, and $\mu(E) = \prod_{k=1}^{K+1} \mu(E_k) \geq (1 - (1 - \bar{\theta})^{n_M})^{K+1}$, which is monotone increasing in M and converges to 1 as $M \to \infty$. □

A.5.3 Proof of Proposition 3

Proof. For $K \geq 2$, $\mu\left(\left\{\{P_\omega\} : \frac{P_\omega(s)}{P_\omega(s')} = \frac{P_{\omega'}(s)}{P_{\omega'}(s)} \text{ for some } P_w, P_{\omega'}, s, s'\right\}\right) = 0$, because the equality restriction defines a lower dimensional set. Now fix $s, s' \in S$ and define the equivalence relation $\sim$ by $P \sim P'$ when $\frac{P_\omega(s)}{P_\omega(s')} > \frac{P_{\omega'}(s)}{P_{\omega'}(s')} \iff \frac{P'_\omega(s)}{P'_\omega(s')} > \frac{P'_{\omega'}(s)}{P'_{\omega'}(s')}$ for all ω, ω'. An equivalence class is denoted by $[P]$. There are M distinct states in Ω and therefore, there are $M!$ distinct equivalence classes, one for each possible strict ordering on the likelihood-ratios, and so $\mu([P]) = \frac{1}{M!}$ for all P. Only two equivalence classes are consistent with the MLRP: $[P]=\{P' : \frac{P'_\omega(s)}{P'_\omega(s')}>\frac{P'_{\omega'}(s)}{P'_{\omega'}(s')}\ \forall v(\omega)>v(\omega')\}$, and $[\hat{P}]=\{P' : \frac{P'_\omega(s)}{P'_\omega(s')}<\frac{P'_{\omega'}(s)}{P'_{\omega'}(s')}\ \forall v(\omega)>v(\omega')\}$. Hence, $\mu\left(\mathcal{P}_M\right) \leq \mu([P]\cup[\hat{P}]) = \frac{2}{M!}$. □

A.5.4 Proof of Proposition 4

Proof. Let c and d be non-trivial inputs. Then, there exists $\omega, \omega', \omega''$, and $\hat{\omega}_c, \tilde{\omega}_c, \hat{\omega}_d, \tilde{\omega}_d$ such that $v(\omega) > v(\omega')$, and $v(\omega) > v(\omega'')$, where $\omega_c = \omega''_c = \hat{\omega}_c$, $\omega_d = \omega'_d = \hat{\omega}_d$, $\omega'_c = \tilde{\omega}_c$, $\omega''_d = \tilde{\omega}_d$, and $\omega_i = \omega'_i = \omega''_i$ for all $i \neq c, d$. We need to consider three cases.

(1) Suppose there are $s_c \in S_c$, $s_d \in S_d$ such that $P_\omega(s_c) > P_{\omega'}(s_c)$ and $P_\omega(s_d) > P_{\omega''}(s_d)$. By condition (2i), $P_\omega(s_d) = P_{\omega'}(s_d)$, and $P_\omega(s_c) = P_{\omega''}(s_c)$. Then, $\frac{P_{\omega''}(s_c)}{P_{\omega''}(s_d)} > \frac{P_\omega(s_c)}{P_\omega(s_d)} > \frac{P_{\omega'}(s_c)}{P_{\omega'}(s_d)}$. Since $v(\omega) > v(\omega')$ and $v(\omega) > v(\omega'')$, the MLRP fails.

(2) Suppose there is no $s_c \in S_c$ such such that $P_\omega(s_c) > P_{\omega'}(s_c)$. Then it must be the case that $P_\omega(s_c) = P_{\omega'}(s_c)$ for all $s_c \in S_c$. By condition (2i), $\frac{P_\omega(s)}{P_\omega(s')} = \frac{P_{\omega'}(s)}{P_{\omega'}(s')}$, for all $s, s' \in S$. Since $v(\omega) > v(\omega')$, the MLRP fails.

(3) Finally, the case where there is no $s_d \in S_d$ such that $P_\omega(s_d) > P_{\omega''}(s_d)$ is analogous to case (2), establishing the result. □

A.5.5 Proof of Proposition 5

Proof. For the probability distribution P on $\Omega \times S$ of a multi-input environment, let $P_{\omega_c^r} \in \Delta_c$ be the conditional distribution on S_c for $\omega_c^r \in \Omega_c$ for $r = 1, ..., M_c$, and let $\gamma_c = P(s \in S_c)$, which is strictly positive generically. In that case, when $K_c \geq M_c$, the system of linear equations $\alpha_c \cdot P_{\omega_c^1} = \frac{\phi(\omega_c^1)}{\gamma_c}$,..., $\alpha_c \cdot P_{\omega_c^{M_c}} = \frac{\phi(\omega_c^{M_c})}{\gamma_c}$ has at least as many unknowns as equations, and therefore generically has a solution $\alpha_c^* \in \mathbb{R}^{K_c}$. When a solution exists, we show that the betweenness property is satisfied. For $c = 1, ..., C$, define $\varphi_c : \Delta \to \mathbb{R}^{K_c}$ by $\varphi_c(x) = (x(s_c^1), ..., x(s_c^{K_c}))$. Define $V(x) \equiv \psi(\alpha_1^* \cdot \varphi_1(x), ..., \alpha_C^* \cdot \varphi_C(x))$. Then V is quasi-linear and monotone in values.[14]

For the converse, without loss of generality, suppose $M_1 > K_1$. Let $\mathcal{P}_1 \equiv \{P_{\omega_1}\}$ be the set of conditional distributions on Δ_1. Following Proposition 1, there is

[14] Suppose $V(x) > w$. For any $\varepsilon > 0$, there is an open neighborhood $N_x(\varepsilon)$ such that $y \in N_x(\varepsilon)$ implies $|\alpha_c^* \cdot \varphi_c(x) - \alpha_c^* \cdot \varphi_c(y)| < \varepsilon$ for $c = 1, .., C$. Since ψ is quasi-linear, $V(y) > 0$ for ε sufficiently small, and so V is lower semicontinuous. Suppose $V(x) \geq V(y)$ and let $z = \theta x + (1-\theta)y$. Then, $\alpha_c^* \cdot \varphi_c(z) = \theta\alpha_c^* \cdot \varphi_c(x) + (1-\theta)\alpha_c^* \cdot \varphi_c(y)$ for $c = 1, ..., C$. Since ψ is quasi-linear, it follows that $V(x) \geq V(z) \geq V(y)$, and so V satisfies betweenness. Finally, for $c = 1, ..., C$, $\varphi_c(P_\omega) = \gamma_c P_{\omega_c}$, and so $\alpha_c^* \cdot \varphi_c(P_\omega) = \gamma_c \alpha_c^* \cdot P_{\omega_c} = \phi(\omega_c)$. Therefore, $V(P_\omega) = \psi(\phi(\omega_1), ..., \phi(\omega_C)) = v(\omega)$, and so V is monotone in values.

$\mathcal{P}_1' \subset \mathcal{P}_1$ with $\mu(\mathcal{P}_1') > 0$ such the conditional for a higher value of input 1 is in the convex hull of the conditionals for lower values. Fix $(\bar{\omega}_2, ..., \bar{\omega}_C)$, and let $\bar{\mathcal{P}} \equiv \{(P_{\omega_1}, P_{\bar{\omega}_2}, ..., P_{\bar{\omega}_C}) : P_{\omega_1} \in \mathcal{P}_1'\}$. By condition (3) in Definition 5, the betweenness property is not satisfied for $P_\omega \in \bar{\mathcal{P}}$. Moreover, $\tilde{\mu}(\bar{\mathcal{P}}) = \gamma_1 \mu(\mathcal{P}_1') > 0$. $\square$

References

Al-Najjar, N. I. (2008): "Large Games and the Law of Large Numbers," *Games and Economic Behavior*, 64, 1–34.

Atakan, A. E. and M. Ekmekci (2014): "Auctions, Actions, and the Failure of Information Aggregation," *American Economic Review*, 104, 2014–48.

Aumann, R. J. (1964): "Markets with a Continuum of Traders," *Econometrica*, 32, 39–50.

Austen-Smith, D. and J. S. Banks (1996): "Information Aggregation, Rationality, and the Condorcet Jury Theorem," *American Political Science Review*, 90, 34–45.

Barelli, P., S. Bhattacharaya, and L. Siga (2019): "Full Information Equivalence in Large Elections," Working Paper.

Bodoh-Creed, A. (2013): "Efficiency and Information Aggregation in Large Uniform-price Auctions," *Journal of Economic Theory*, 148, 2436–2466.

Condorcet, M. d. (1785): "Essai sur l'Application de l'Analyse a la Probabilité des Decisions Rendues a la Pluralité des Voix." Trans. McLean Iain and Hewitt Fiona, 1994. Paris.

de Castro, L. (2010): "Affiliation, Equilibrium Existence and Revenue Ranking of Auctions," Discussion Paper No. 1530, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Feddersen, T. J. and W. Pesendorfer (1997): "Voting Behavior and Information Aggregation in Elections with Private Information," *Econometrica*, 65, 1029–1058.

Grossman, S. and J. Stiglitz (1976): "Information and Competitive Price Systems," *American Economic Review*, 66, 246–53.

Harstad, R. M., A. S. Pekeč, and I. Tsetlin (2008): "Information Aggregation in Auctions with an Unknown Number of Bidders," *Games and Economic Behavior*, 62, 476–508.

Hellwig, M. F. (1980): "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22, 477–498.

Jackson, M. (2003): "Efficiency and Information Aggregation in Auctions with Costly Information," *Review of Economic Design*, 8, 121–141.

Kremer, I. (2002): "Information Aggregation in Common Value Auctions," *Econometrica*, 70, 1675–1682.

Kyle, A. S. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53, 1315–1335.

——— (1989): "Informed Speculation with Imperfect Competition," *The Review of Economic Studies*, 56, 317–355.

Lambert, N. S., M. Ostrovsky, and M. Panov (2018): "Strategic Trading in Informationally Complex Environments," *Econometrica*, 86, 1119–1157.

Lauermann, S. and A. Wolinsky (2017): "Bidder Solicitation, Adverse Selection, and the Failure of Competition," *American Economic Review*, 107, 1399–1429.

Mihm, M. and L. Siga (2020a): "A Behavioral Characterization of the Likelihood Ratio Order," Available at SSRN: http://ssrn.com/abstract=3524753.

——— (2020b): "Competitive Prices in a Large Market with Private Information," Available at SSRN: http://ssrn.com/abstract=3628672.

Milgrom, P. R. (1979): "A Convergence Theorem for Competitive Bidding with Differential Information," *Econometrica*, 47, 679–88.

——— (1981): "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, 49, 921–43.

Milgrom, P. R. and R. J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089–1122.

Palfrey, T. R. (1985): "Uncertainty Resolution, Private Information Aggregation and the Cournot Competitive Limit," *The Review of Economic Studies*, 52, 69–83.

Pesendorfer, W. and J. M. Swinkels (1997): "The Loser's Curse and Information Aggregation in Common Value Auctions," *Econometrica*, 65, 1247–1282.

——— (2000): "Efficiency and Information Aggregation in Auctions," *The American Economic Review*, 90, 499–525.

Radner, R. (1979): "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices," *Econometrica*, 47, 655–678.

Reny, P. J. and M. Perry (2006): "Toward a Strategic Foundation for Rational Expectations Equilibrium," *Econometrica*, 74, 1231–1269.

Serrano-Padial, R. (2012): "Naive Traders and Mispricing in Prediction Markets," *Journal of Economic Theory*, 147, 1882–1912.

Vives, X. (2011): "Strategic Supply Function Competition With Private Information," *Econometrica*, 79, 1919–1966.

——— (2014): "On the Possibility of Informationally Efficient Markets," *Journal of the European Economic Association*, 12, 1200–1239.

Wilson, R. (1977): "A Bidding Model of Perfect Competition," *Review of Economic Studies*, 44, 511–18.

Wolinsky, A. (1990): "Information Revelation in a Market with Pairwise Meetings," *Econometrica*, 58, 1–23.