A Behavioral Characterization of the Likelihood Ratio Order*

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Abstract

It is well-known that stochastic dominance is equivalent to a unanimity property for monotone expected utilities. For lotteries over a finite set of prizes, we establish an analogous relationship between likelihood-ratio dominance and monotone betweenness preferences, which are an important generalization of expected utility.

Key words: betweenness preferences, expected utility, likelihood-ratio dominance, stochastic dominance.

1 Introduction

Consider the set of lotteries over a finite set of monetary prizes. In a seminal paper, Quirk and Saposnik (1962) show that lottery p (first-order) stochastically dominates lottery q if and only if *every* monotone expected utility function assigns a higher utility to p than q.[1] The significance of this result is two-fold. First, it provides a behavioral interpretation of stochastic dominance, which is a prominent stochastic order in probability and statistics. Second, it facilitates non-parametric predictions for behavior under risk, as commonly encountered in economics and finance: if more money is preferred to less, stochastic dominance characterizes the observable implications of the expected utility hypothesis.

In this paper, we establish an analogous relationship between *likelihood-ratio dominance*, which is another prominent stochastic order in probability and statistics, and *betweenness*

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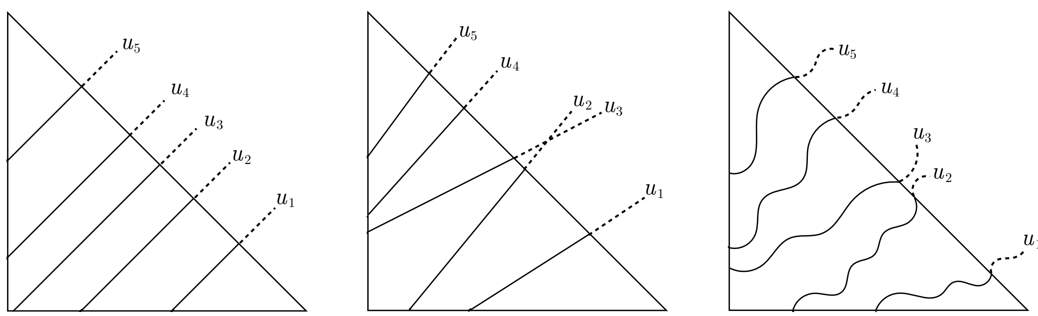
[1] In another seminal paper, Hadar and Russell (1969) show that the characterization of stochastic dominance also holds for continuous random variables. Similar to Quirk and Saposnik (1962), we focus on settings with a finite set of prizes in this paper.

preferences, which are an important generalization of expected utility introduced in Chew (1983) and Dekel (1986).

Betweenness preferences are characterized by weakening the controversial independence axiom and can thereby accommodate widely-documented violations of the expected utility hypothesis (e.g., the Allais, 1953, paradox). Similar to expected utility, the indifference curves of a betweenness preference are linear but, unlike expected utility, are not necessarily parallel (Figure 1). As such, betweenness preferences can remedy descriptive failures of the expected utility hypothesis while maintaining both quasiconvexity and quasiconcavity. These properties are analytically attractive because, for instance, quasiconcavity is required to establish existence of Nash equilibrium, while quasiconvexity is necessary for dynamic consistency in intertemporal choice (see, e.g., Dekel, 1986).



(a) Expected utility (b) Betweenness (c) Non-betweenness

Figure 1: Indifference curves for preferences over lotteries.

As with the classic relationship between stochastic dominance and expected utility, our characterization serves a dual purpose. For decision analysis, it provides a simple criterion for making robust predictions across the whole class of betweenness preferences based on a well-known stochastic order from probability and statistics. Conversely, the characterization provides a behavioral foundation for the likelihood-ratio order that may prove useful when interpreting results from information economics and mechanism-design, where the likelihood-ratio order is often a central assumption. We discuss these interpretations in more detail after presenting the result.

The paper is organized as follows. Section 2 describes monotone betweenness preferences in terms of their axiomatic properties. Section 3 recalls the stochastic and likelihood-ratio orders, and illustrates the connection to monotone preferences geometrically. In Section 4, we state, prove and discuss the characterization result.

2 Betweenness preferences

Let Δ be the set of probability distributions over a finite set X. Typical elements of X are denoted x, y, z and called *prizes*. Typical elements of Δ are denoted p, q, r and called *lotteries*. We denote by $p(x)$ the probability that lottery p assigns to prize x, and let δ_x be the lottery that assigns probability 1 to prize x. The set of prizes is endowed with a total order $\trianglerighteq$, which is a primitive ranking of the prizes. For instance, if prizes are monetary, it may be natural to define $x \trianglerighteq y$ if and only if $x \geq y$.

Let $\succeq$ be a binary relation on the set of lotteries, with asymmetric part $\succ$ and symmetric part $\sim$. The following axioms are standard in the literature on decision-making under risk, where $\succeq$ is interpreted as a preference relation over lotteries.

Axiom 1 (Weak order)**.** For all $p, q, r \in \Delta$: (i) $p \succeq q$ or $q \succeq p$, and (ii) $p \succeq q$ and $q \succeq r$ implies $p \succeq r$.

Axiom 2 (Continuity)**.** If $p \succ q \succ r$, then $\theta p + (1-\theta)r \sim q$ for some $\theta \in (0,1)$.

Axiom 3 (Non-triviality)**.** There are lotteries p and q such that $p \succ q$.

The binary relation $\succeq$ has an expected utility representation if and only if, in addition to Axioms 1–3, $\succeq$ satisfies the independence axiom:

Axiom 4 (Independence)**.** For all $\theta \in (0,1)$, $p \succeq q$ implies $\theta p + (1-\theta)r \succeq \theta q + (1-\theta)r$.

Motivated by empirical violations of the independence axiom, Dekel (1986) proposes the following generalization:

Axiom 5 (Betweenness)**.** For all $\theta \in (0,1)$, (i) $p \succ q$ implies $p \succ \theta p + (1-\theta)q \succ q$, and (ii) $p \sim q$ implies $p \sim \theta p + (1-\theta)q \sim q$.

It is easily verified that independence implies betweenness but not vice versa. The generalization is consequential because the betweenness axiom is consistent with many behavioral phenomena that are precluded by the independence axiom (see, e.g., Machina, 1982; Chew, 1983; Dekel, 1986; Gul, 1991; Starmer, 2000; Siga and Mihm, 2020; Cerreia-Vioglio et al., 2020).

Definition 1. The binary relation $\succeq$ is (i) a *linear preference* if it satisfies Axioms 1–4, and (ii) a *betweenness preference* if it satisfies Axioms 1–3 and Axiom 5.

We require one further property of the preference relation: a betweenness/linear preference is *monotone* if a greater prize (for sure) is preferred to a lesser prize (for sure):

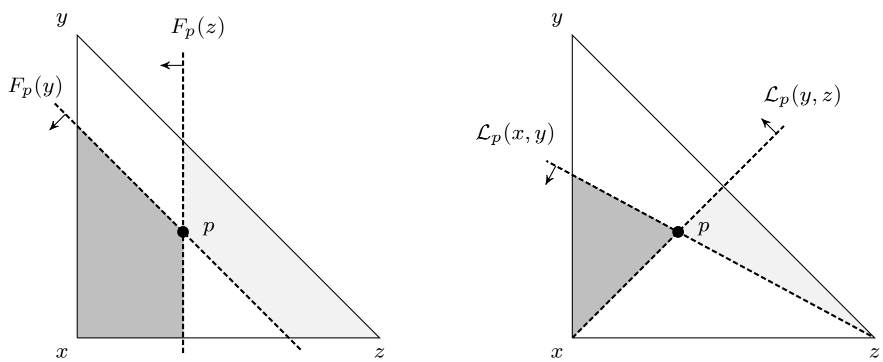
Axiom 6 (Monotonicity)**.** $x \trianglerighteq y$ implies $\delta_x \succeq \delta_y$.

3 Stochastic orders

We recall two stochastic orders over lotteries, which occupy a central role in probability and statistics (see, e.g., Shaked and Shanthikumar, 1994). To simplify arguments involving likelihood-ratios, we focus on the set of lotteries with full support, denoted $\tilde{\Delta}$. For a lottery $p \in \tilde{\Delta}$, let $F_p(x) \equiv \sum_{y \trianglelefteq x} p(y)$ be the cumulative distribution function over prizes, and $\mathcal{L}_p(x, y) \equiv \frac{p(x)}{p(y)}$ be the likelihood-ratio function.

Definition 2. For lotteries $p, q \in \tilde{\Delta}$, (i) p *stochastically dominates* q if $F_p(x) \leq F_q(x)$ for all prizes x, and (ii) p *likelihood-ratio dominates* q if $\mathcal{L}_p(x, y) \geq \mathcal{L}_q(x, y)$ for all prizes $x \trianglerighteq y$.

To illustrate, suppose there are three prizes, $x \trianglerighteq y \trianglerighteq z$, so that lottery p can be depicted as a point in the unit simplex.



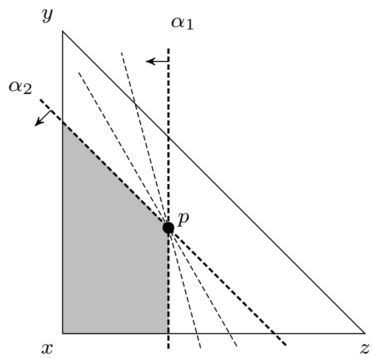
(a) Stochastic dominance (b) Likelihood-ratio dominance

Figure 2: Geometric illustration of stochastic orders.

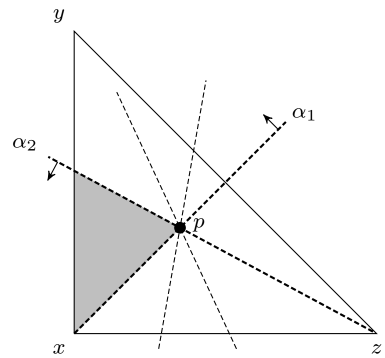
In Figure 2a, any lottery q on the line $F_p(z)$ satisfies $F_q(z) = F_p(z)$, while any lottery r on the line $F_p(y)$ satisfies $F_r(y) = F_p(y)$. As a result, the dark region represents lotteries that stochastically dominate p; the light region represents lotteries that are stochastically dominated by p; and remaining lotteries are stochastically non-comparable to p.

In Figure 2b, any lottery q on the line $\mathcal{L}_p(y, z)$ satisfies $\mathcal{L}_q(y, z) = \mathcal{L}_p(y, z)$, while any lottery r on the line $\mathcal{L}_p(x, y)$ satisfies $\mathcal{L}_r(x, y) = \mathcal{L}_p(x, y)$. The dark region therefore represents lotteries that likelihood-ratio dominate p; the light region represents lotteries that are likelihood-ratio dominated by p; and remaining lotteries are likelihood-ratio non-comparable to p. The dark (resp. light) region in Figure 2b is a strict subset of the dark (resp. light) region in Figure 2a, reflecting that likelihood-ratio dominance implies stochastic dominance.

The following figure illustrates the connection to monotone preferences, and provides much of the intuition for our result.



(a) Linear preferences



(b) Betweenness preferences

Figure 3: Geometric illustration of monotone preferences.

Figure 3a illustrates potential indifference curves containing lottery p for monotone linear preferences. Without monotonicity, any straight line can represent an indifference curve for some linear preference, but monotonicity imposes additional restrictions. Line α_1 represents an indifference curve of a monotone linear preference, as long as the upper contour set is to the left. A clockwise rotation would not represent an indifference curve because a translation has δ_y in the upper and δ_x in the lower contour set (violating $\delta_x \succeq \delta_y$). Rotating anti-clockwise yields other potential indifference curves but beyond line α_2, translations will have δ_y in the lower and δ_z in the upper contour set (violating $\delta_y \succeq \delta_z$). As a result, only straight lines between α_1 and α_2 can represent indifference curves of a monotone linear preference. The shaded region therefore represents lotteries that are preferred to p for *every* monotone linear preference, which coincides with the dark region in Figure 2a.

For a betweenness preference, indifference curves are also straight lines but do not need to be parallel, which implies that there is a larger set of potential indifference curve. In Figure 3b, line α_1 cannot be an indifference curve for a monotone betweenness preference because it contains δ_x but has δ_y in the upper contour set (violating $\delta_x \succeq \delta_y$). However, rotating anti-clockwise yields a potential indifference curve because, while a translation would have δ_y in the upper and δ_x in the lower contour set, higher indifference curves need not be parallel. As long as the line passing through p has both δ_y and δ_x in the upper contour set, it is possible to complete the map of indifference curves to represent a monotone betweenness preference (see Lemma 3). Again, one cannot rotate too far because line α_2 contains δ_z but has δ_y in the lower contour set (violating $\delta_y \succeq \delta_z$). The shaded region in Figure 3b therefore represents lotteries that are preferred to p for *every* monotone betweenness preference, which coincides with the dark region in Figure 2b.

4 Characterization

The intuition from Figures 2a and 3a extends to more than three prizes: lottery p stochastically dominates lottery q if and only if $p \succeq q$ for every monotone linear preference. We establish an analogous relationship between the likelihood-ratio order and monotone betweenness preferences.

Theorem 1. *Lottery p likelihood-ratio dominates lottery q if and only if $p \succeq q$ for every monotone betweenness preference $\succeq$.*

As Figures 2b and 3b suggest, the proof is geometric. In Lemma 1, we first characterize likelihood-ratio dominance in terms of a single-crossing property of supporting hyperplanes. In Lemma 2, we then invoke a result from Siga and Mihm (2020) to show that single-crossing also characterizes monotonicity for betweenness preferences. Finally, Lemma 3 shows how to complete a partial map of indifference curves for a monotone betweenness preference. Theorem 1 follows immediately from these arguments.

Notation: Without loss of generality, enumerate prizes $X = \{x_1, ..., x_K\}$ so that $x_k \trianglelefteq x_{k+1}$ for $k = 1, .., K-1$. In the following, all vectors are in $\mathbb{R}^K$ and we denote by $\alpha(k)$ the k-th component of vector α. We identify Δ with the unit simplex on $\mathbb{R}^K$ and set $p(k) \equiv p(x_k)$ and $\delta_k \equiv \delta_{x_k}$. For a vector α, define the absolute value vector $|\alpha|$ by $|\alpha|(k) \equiv |\alpha(k)|$ for $k = 1, ..., K$.

We say that a vector α satisfies *single-crossing* if $\alpha(k+1) \leq 0$ implies $\alpha(k) \leq 0$ for $k = 1, ..., K-1$. For lottery p, let $\mathcal{C}(p)$ be the set of non-zero vectors α that satisfy single-crossing and $\alpha \cdot p = 0$.

Likelihood-ratio dominance and single-crossing: The following lemma provides a geometric characterization of likelihood-ratio dominance.

Lemma 1. *Lottery p likelihood-ratio dominates q if and only if $\alpha \cdot p \geq 0$ for all $\alpha \in \mathcal{C}(q)$.*

Proof. Suppose p likelihood-ratio dominates q. For $\alpha \in \mathcal{C}(q)$, let $k^* = \max\{k : \alpha(k) \leq 0\}$ (k^* is well-defined because $\alpha \cdot q = 0$). Since p likelihood-ratio dominates q, $q(k) \geq \frac{q(k^*)}{p(k^*)} p(k)$ for $k \leq k^*$ and $q(k) \leq \frac{q(k^*)}{p(k^*)} p(k)$ for $k > k^*$. Since α satisfies single-crossing, $\alpha(k) \leq 0$ for $k \leq k^*$, and $\alpha(k) > 0$ for $k > k^*$. Therefore,

$$0 = \alpha \cdot q \leq \sum_{k=1}^{k^*} \alpha(k) \left(\frac{q(k^*)}{p(k^*)} p(k) \right) + \sum_{k=k^*+1}^{K} \alpha(k) \left(\frac{q(k^*)}{p(k^*)} p(k) \right) = \frac{q(k^*)}{p(k^*)} \alpha \cdot p.$$

Since $\frac{q(k^*)}{p(k^*)} > 0$ and $\alpha \in \mathcal{C}(q)$ was arbitrary, $\alpha \cdot p \geq 0$ for all $\alpha \in \mathcal{C}(q)$.

For the converse, suppose $\alpha \cdot p \geq 0$ for all $\alpha \in \mathcal{C}(q)$. Fix $n, m = 1, ..., K$ such that $n > m$. Given n and m with $n > m$, for $\varepsilon \in (0,1)$ define $\alpha_\varepsilon \in \mathbb{R}^K$ by

$$\alpha_\varepsilon(k) = \begin{cases} -(1-\varepsilon)q(n) - \varepsilon \sum\limits_{i=n+1}^{K} q(i) & \text{if } k = m \\ (1-\varepsilon)q(m) & \text{if } k = n \\ \varepsilon\, q(m) & \text{if } k > n \\ 0 & \text{otherwise} \end{cases}$$

Then α_ε satisfies single-crossing and $\alpha_\varepsilon \cdot q = 0$, hence $\alpha_\varepsilon \in \mathcal{C}(q)$. Therefore,

$$0 \leq \alpha_\varepsilon \cdot p = (1-\varepsilon)\Big(q(m)p(n) - q(n)p(m)\Big) + \varepsilon\left(q(m)\sum_{i=n+1}^{K} p(i) - p(m)\sum_{i=n+1}^{K} q(i)\right).$$

The inequality holds for all $\varepsilon \in (0,1)$ and so $\frac{p(n)}{p(m)} \geq \frac{q(n)}{q(m)}$. Since $n > m$ were arbitrary, p likelihood-ratio dominates q. $\square$

Monotone betweenness preferences and single-crossing: A binary relation $\succeq$ on Δ is represented by a function $V : \Delta \to \mathbb{R}$ if, for all $p, q \in \Delta$, $p \succeq q \quad \Leftrightarrow \quad V(p) \geq V(q)$. The function V satisfies betweenness if, for all $\theta \in (0,1)$ and $p, q \in \Delta$, the following conditions are satisfied (i) $V(p) > V(q)$ implies $V(p) > V(\theta p + (1-\theta)q) > V(q)$, and (ii) $V(p) = V(q)$ implies $V(p) = V(\theta p + (1-\theta)q) = V(q)$. By standard arguments (see, e.g., Dekel, 1986, Proposition A.1), a binary relation $\succeq$ on Δ is a betweenness preference if and only if it is represented by a non-constant continuous function $V : \Delta \to \mathbb{R}$ that satisfies betweenness.

Since V is continuous and Δ is compact, $V(\Delta)$ is a compact interval, and betweenness implies that V is quasi-linear (i.e., both quasi-concave and quasi-convex). As a result, Lemma 4 in Siga and Mihm (2020) shows that any betweenness preference $\succeq$ also has a *vector-representation*: there is a collection of non-zero vectors $\{\alpha_p\}_{p\in\Delta}$ such that $q \succsim\!\!\!\!\precsim p$ if and only if $\alpha_p \cdot q \gtreqless 0$; morevoer, $q \succsim\!\!\!\!\precsim p$ if and only if $\alpha_p \gtreqless \alpha_q$, so that $(\{\alpha_p\}_{p\in\Delta}, \geq)$ is a chain with the standard vector order $\geq$ on $\mathbb{R}^K$.[2] The following lemma to shows that single-crossing characterizes the additional implications of monotonicity (Axiom 6) for the vector-representation of a betweenness preference.

Lemma 2. *Let $\succeq$ be any betweenness preference, and let $\{\alpha_p\}_{p\in\Delta}$ be its vector-representation. Then, $\succeq$ satisfies Axiom 6 if and only if α_p satisfies single-crossing for all lotteries p.*

[2]By the equivalence $[q \succsim\!\!\!\!\precsim p \;\Leftrightarrow\; \alpha_p \cdot q \gtreqless 0]$ we mean that (i) $q \succ p$ if and only if $\alpha_p \cdot q > 0$, (ii) $q \sim p$ if and only if $\alpha_p \cdot q = 0$, and (iii) $q \prec p$ if and only if $\alpha_p \cdot q < 0$. Similarly, for the equivalence $[q \succsim\!\!\!\!\precsim p \;\Leftrightarrow\; \alpha_p \gtreqless \alpha_q]$ we mean that (i) $q \succ p$ if and only if $\alpha_p > \alpha_q$, (ii) $q \sim p$ if and only if $\alpha_p = \alpha_q$, and (iii) $q \prec p$ if and only if $\alpha_p < \alpha_q$, where $\alpha > \alpha'$ means $\alpha \geq \alpha'$ and $\alpha \neq \alpha'$.

Proof. Suppose that $\succeq$ satisfies Axiom 6 and consider any lottery p such that $\alpha_p(k) \leq 0$. Then, $\alpha_p \cdot \delta_k \leq 0$ and therefore $p \succeq \delta_k$. By Axiom 6, $p \succeq \delta_{k-1}$, and hence $\alpha_{k-1} \geq \alpha_p$. Since $\alpha_{\delta_{k-1}} \cdot \delta_{k-1} = 0$, $\alpha_{\delta_{k-1}}(k-1) = 0$, and therefore $\alpha_p(k-1) \leq 0$. Since k was arbitrary, α_p satisfies single-crossing.

For the converse, suppose α_p is single-crossing for all p and consider indeces $n > m$. Since $\alpha_{\delta_n}\delta_n = 0$, $\alpha_{\delta_n}(n) = 0$. By single-crossing, $\alpha_{\delta_n}(m) \leq 0$, and therefore $\alpha_{\delta_n}\delta_m \leq 0$, which implies $\delta_n \succeq \delta_m$. Since $n > m$ were arbitrary, $\succeq$ satisfies Axiom 6. $\square$

Finally, the following lemma provides one way to complete the map of indifference curves for a monotone betweenness preference given some initial indifference curve satisfying single-crossing.

Lemma 3. *If lottery p^* has full support and $\alpha \in \mathcal{C}(p^*)$, then there is a monotone betweenness preference $\succeq$ such that $q \underset{\precsim}{\succsim} p^*$ if and only if $\alpha \cdot q \gtreqless 0$.*

Proof. For lottery p such that $\alpha \cdot p = 0$, let $\alpha_p \equiv \alpha$. For lottery p' such that $\alpha \cdot p' > 0$, there exists a unique $\lambda_{p'} \in (0,1)$ such that $\lambda_{p'}(\alpha \cdot p') + (1-\lambda_{p'})(-|\alpha| \cdot p') = 0$, since $-|\alpha| \cdot p' < 0$. In that case, let $\alpha_{p'} \equiv \lambda_{p'}\alpha - (1-\lambda_{p'})(|\alpha|)$. For lottery p'' such that $\alpha \cdot p < 0$, there exists a unique $\lambda_{p''} \in (0,1)$ such that $\lambda_{p''}(\alpha \cdot p'') + (1-\lambda_{p''})(|\alpha| \cdot p'') = 0$, since $|\alpha| \cdot p'' > 0$. In that case, let $\alpha_{p''} \equiv \lambda_{p''}\alpha + (1-\lambda_{p''})(|\alpha|)$. We thereby obtain non-zero vectors $\{\alpha_p\}_{p\in\Delta}$, which (by construction) form a chain when paired with the standard vector order $\geq$ on $\mathbb{R}^K$.

Now define the binary relation $\succeq$ on Δ by $q \succeq p$ if and only if $\alpha_p \cdot q \geq 0$. By construction, (i) $q \succeq p$ if and only if $\alpha_p \geq \alpha_q$, (ii) the mapping $p \mapsto \alpha_p$ is continuous, (iii) $\alpha_{\delta_1} \cdot \delta_K > 0$, (iv) $p \underset{\precsim}{\succsim} q$ implies $p \underset{\precsim}{\succsim} \theta p + (1-\theta) q \underset{\precsim}{\succsim} q$ for $\theta \in (0,1)$, and (v) α_p satisfies single-crossing for all p. It is then straightforward to show that property (i) implies that $\succeq$ satisfies Axiom 1, because $(\{\alpha_p\}_{p\in\Delta}, \geq)$ is a chain; property (ii) implies Axiom 2; property (iii) implies Axiom 3; property (iv) implies Axiom 5; and property (v) implies Axiom 6. Therefore, $\succeq$ is a monotone betweenness preference such that $q \underset{\precsim}{\succsim} p^*$ if and only if $\alpha \cdot q \gtreqless 0$. $\square$

Proof of Theorem 1: The proof follows directly from Lemma 1–3, and we argue the contrapositive for each direction. Let $p, q \in \tilde{\Delta}$, and suppose p does not likelihood-ratio dominate q. By Lemma 1, there is $\alpha \in \mathcal{C}(q)$ such that $\alpha \cdot p < 0$. Hence, by Lemma 3, we can construct a monotone betweenness preference $\succeq$ such that $q \succ p$. For the converse, suppose there is a monotone betweenness preference $\succeq$ such that $q \succ p$. Using the vector-representation in Lemma 2, $\alpha_q \cdot p < 0$ and $\alpha_q \in \mathcal{C}(q)$. Hence, by Lemma 1, p does not likelihood-ratio dominate q. $\square$

Discussion

Theorem 1 establishes a direct connection between the likelihood-ratio order and an economically meaningful class of preferences, which occupy a central role in the literature on decision making under risk.[3]

On one hand, the characterization in terms of betweenness preferences provides new insights on the likelihood-ratio order, which has many applications in mechanism design and information economics. For instance, consider a standard model of information, where signals provide information about an unknown outcome of interest. The information structure is said to satisfy the *monotone likelihood-ratio property (MLRP)* if outcomes and signals are ordered such that $P(.|s)$ likelihood-ratio dominates $P(.|s')$ whenever $s \trianglerighteq s'$, where $P(.|.)$ is the conditional distribution over outcomes given a signal. Milgrom (1981) introduced the MLRP for problems in information economics and mechanism design to formalize the idea that signals convey more or less favorable news about outcomes, because higher signals induce uniformly more optimistic posteriors. Theorem 1 clarifies the behavioral meaning of the intuitive idea that signals convey more or less favorable news: information satisfies the MLRP if and only if any decision maker with monotone betweenness preferences prefers the conditional distribution over outcomes given signal s to the conditional distribution given signal s'. For economic applications, Theorem 1 therefore identifies the domain of preferences where the MLRP guarantees an unambiguous favorability ranking over signals. Unlike stochastic dominance, the favorability ranking extends to non-expected utility preferences; however, beyond the class of betweenness preferences, decision makers will disagree about the favorability of signals, and so the MLRP no longer reflects the idea that one signal conveys better news than another.

An understanding of the behavioral implications of the likelihood-ratio order seems important because, along with stochastic dominance, likelihood-ratio dominance is arguably the most prominent stochastic order in economic applications. As a corollary, Theorem 1 also has implications for other stochastic orders in probability and statistics, such as the hazard-rate and reverse hazard-rate order (see, e.g., Shaked and Shanthikumar, 1994, Chapter 1). As likelihood-ratio dominance implies both hazard-rate and reverse hazard-rate dominance, which in turn imply stochastic dominance, these stochastic orders must be characterized by a class of monotone preferences that is larger than expected utility but smaller than the class of betweenness preferences.

[3] Lemma 1 also provides a single-crossing characterization of likelihood-ratio dominance for lotteries on a finite domain. In different contexts from ours, a connection between ratio orders and single-crossing properties has been exploited in the prior literature. For instance, Kartik et al. (2019) show that a linear combination of vectors has a single-crossing property if and only if each vector individually has the single-crossing property and the vectors are ratio-ordered (see also Quah and Strulovici, 2012).

On the other hand, Theorem 1 provides a non-parametric prediction for behavior under risk. When more money is preferred to less, stochastic dominance provides a simple testable prediction of the expected utility hypothesis. Betweenness preferences provide a tractable remedy for some of the descriptive failures of expected utility, and Theorem 1 identifies the additional admissible behaviors: choices that are inconsistent with stochastic dominance but respect likelihood-ratio dominance. In particular, choices that are inconsistent with the likelihood ratio order are also inconsistent with monotone betweenness preferences. Our characterization of the likelihood-ratio order therefore offers a simple testable prediction for an important class of non-expected utility preferences.

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